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Preface

Optical wave propagation through random media is a fairly mature subject area, having been studied extensively since the 1950s. Consequently, a number of research monographs and textbooks on optical wave propagation have emerged over the years, including *Laser Beam Propagation through Random Media*, by two of the current authors (Andrews and Phillips). Thus far, the general theory is well developed for weak fluctuation regimes using the Rytov method—i.e., tractable expressions have been derived for most of the statistical quantities of interest, including those for a simple Gaussian-beam wave. Theory for strong fluctuations has also produced tractable results for certain second-order field statistics, based on the extended Huygens-Fresnel principle, the parabolic equation method, or other methods. Still, there are problems of great interest where adequate theory and useful analytic expressions have not evolved in a satisfactory way. This is the case, for example, for *optical scintillation*—a fourth-order field statistic that describes the irradiance fluctuations of an optical wave. Optical scintillation is considered one of the most crucial atmospheric effects that must be fully understood because it ultimately determines the performance limitation of an optical system.

In most applications the irradiance fluctuations usually range from *weak fluctuations* (generally associated with shorter path lengths) up to the point where scintillation attains its peak values, the so-called *focusing regime*. Because of the potential for large values of scintillation, the focusing regime can be considered the most hostile to optical systems; unfortunately, the rigorously developed scintillation theory is not applicable in this regime. Applications that involve very long propagation paths may cause the irradiance fluctuations to extend beyond the focusing regime into the *saturation regime* where scintillation begins to decrease toward a limiting value of unity. Asymptotic expressions (including inner scale effects) have been derived for the saturation regime, but recent findings show that these results do not compare well with either experimental or simulation data.

The purpose of this book on laser beam scintillation is twofold—first, to present a tractable theory of optical scintillation in the atmosphere that is applicable under all irradiance fluctuation conditions, and second, to investigate the impact of optical scintillation on system performance in the application areas of free-space laser communication (i.e., *lasercom*) and laser radar. Renewed interest in laser communications, laser radar, and other application areas involving beam wave propagation through atmospheric turbulence has provided the impetus in recent years for developing several new and useful analytic models of scintillation behavior that extend from weak-to-strong irradiance fluctuations. This book is an attempt to connect these new models with other well-known theoretical expressions, and with experimental and simulation data. Although the new models arise from a heuristic theory of scintillation, the theory is based on sound principles of physics embraced over the years by many researchers. Moreover, the ensuing analytic scintillation models are simple and can be used for most propagation circumstances. Perhaps one of the more interesting aspects of this theory is the fact that it predicts significant effects from the presence of a finite outer scale under moderate-to-strong irradiance fluctuations, causing a steeper drop in scintillation beyond the focusing regime than that predicted by the conventional asymptotic theory. Outer scale effects on scintillation are contrary to traditional beliefs that are fostered primarily by weak fluctuation theories. Where possible, we make quantitative and qualitative comparisons of scintillation results with published experimental and simulation data in order to lend support to the scintillation models developed here.

We have broadly divided the book into two parts—background and general *scintillation models* are developed in Part I (Chapters 1–6) and *applications* of optical wave propagation are discussed in Part II (Chapters 7–10). The intended audience for this text includes practicing engineers and scientists who are interested in a sound understanding of propagation phenomena and the role of scintillation on optical system behavior. In that regard the material contained within should aid design engineers in identifying some of the limitations on system performance imposed by the atmosphere.

In Chapter 1 we provide a brief review of basic optical wave propagation issues and provide many well-known theoretical results, most of which are based on weak fluctuation theory. Derivations of the analytic results are generally not given in this first chapter, but appropriate references are cited for more detailed discussions and derivations. Chapter 2 presents the physical concepts upon which the heuristic theory of scintillation is based. Specifically, we introduce the *modified Rytov theory* which invokes the premise that, within the distribution of turbulent scale sizes (inhomogeneities) in the atmosphere, those with dimensions between the spatial coherence radius and the scattering disk of the optical wave do not contribute significantly to scintillation in moderate-to-strong fluctuation regimes. Therefore, to eliminate such scale size effects from the analysis, we formally introduce appropriate filter functions into the spatial power spectrum of refractive-index fluctuations. Current probability distribution models proposed for optical wave propagation are also reviewed in this second chapter. A systematic development of the heuristic theory is then presented in Chapters 3 through 5 for infinite plane waves, spherical waves, and Gaussian-beam waves, respectively. Because of their inherent simplicity over Gaussian-beam waves, we use the plane wave and spherical wave developments in Chapters 3 and 4 as "stepping stones" to the more useful results in Chapter 5 concerning Gaussian-beam waves. Our analysis in all cases leads to tractable models for large-scale and small-scale scintillations, based only on knowledge of the atmospheric refractive-index structure parameter C_n^2, propagation path length L, inner scale of turbulence l_0, and outer scale of turbulence L_0, in addition to various beam characteristics like wavelength, beam spot size, and phase front radius of curvature. Each type of optical wave leads to different cutoff spatial frequencies (or wave number) for the large-scale and small-scale filters for the turbulent eddy cell distribution. The scintillation theory is then extended in Chapter 6 to large aperture receiving systems that reduce the scintillation through a process known as *aperture averaging*.

Part II on applications starts with Chapter 7 and a review of optical communication systems, analyzing the distinctions between direct detection and coherent detection receiving systems. In particular, we include an analysis of scintillation effects on system performance in regards to signal-to-noise ratio (SNR) and bit error rate (BER). Mitigating techniques for optical scintillation are discussed in terms of spatial diversity architectures. The treatment of optical communication systems is followed in Chapter 8 by an analysis of fade statistics associated with various optical channels. The optical channels of interest involve line-of-sight terrestrial links as well as uplink/downlink channels for satellite communication. Fade probabilities predicted by both the gamma-gamma distribution and lognormal distribution are presented and compared for these channels. We consider the gamma-gamma distribution a general irradiance model for both weak and strong fluctuations.

In Chapter 9, the double-pass propagation phenomenon associated with laser radar systems is analyzed primarily in terms of scintillation statistics under general conditions of irradiance fluctuations. The increase in mean irradiance along the optical axis of a monostatic system, often referred to as the *enhanced backscatter effect*, is also discussed. Target models examined in this analysis are a small unresolved target (also called a "point target") and a finite diffuse target. Models for threshold detection are presented, and experimental data collected for an eight-element equal-gain coherent detection array are compared with the double-pass theoretical models developed here. Lastly, in Chapter 10 we examine some of the impact of scintillation on incoherent imaging systems. Linear system theory is briefly reviewed for both coherent and incoherent systems, the latter presented in the context of a laser imaging radar. We end this chapter with a discussion of the role of optical scintillation and target speckle on performance characteristics like target resolution and single pixel signal-to-noise ratio.

Finally, we wish to extend our appreciation to M. Ammar Al-Habash, who participated as a coauthor on several key papers providing the mathematical foundation for this textbook. His physical insights and useful suggestions were most helpful to us during many long discussions at the blackboard.

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Symbols and Notation

BER	Bit error-rate
$B_I(\mathbf{r}_1,\mathbf{r}_2,L)$, $B_I(\rho,L)$	Covariance function of irradiance
$B_I^{ie}(\mathbf{r},L)$, $B_{\ln I}^{ie}(\mathbf{r},L)$	Correlation functions associated with amplitude enhancement of reflected wave
$B_x(\rho)$, $B_y(\rho)$	Large-scale and small-scale covariances
$B_{\ln x}(\rho)$, $B_{\ln y}(\rho)$	Large-scale and small-scale log covariances
$b_I(\rho,L)$	Normalized covariance function of irradiance
$C(\mathbf{r})$	Random amplification factor of irradiance
C_n^2, $C_n^2(h)$	Refractive-index structure parameter
CNR	Carrier-to-noise ratio
CTF	Coherent transfer function
c	Speed of light (=3 × 10^8 m/s)
D, D_G	Diameter of collecting lens
$D(\mathbf{r}_1,\mathbf{r}_2,L)$, $D(\rho,L)$	Wave structure function
DOC	Modulus of the complex degree of coherence
$D_n(R)$	Index of refraction structure function
EG	Equal gain coherent detection scheme
EO	Electro-optics
$\mathrm{erf}(x)$, $\mathrm{erfc}(x)$	Error functions
$E_n(\mathbf{r}_1,\mathbf{r}_2)$, $n=1,2,3$	Second-order moments of the complex phase perturbations
EBS	Enhanced backscatter
FAR	False alarm rate
FOV	Field of view
F_G	Effective focal length of Gaussian lens
F_T	Fade level (in dB) below the mean on-axis irradiance
F_0	Phase front radius of curvature of beam at transmitter

F	Phase front radius of curvature of beam at receiver
${}_pF_q$	Generalized hypergeometric function
$G(\mathbf{s},\mathbf{r};L)$	Green's function
$G_x(\kappa,l_0)$, $G_y(\kappa)$	Large-scale and small-scale filter functions
$\hat{H}(\mathbf{v})$	Coherent transfer function (CTF)
$I^0(\mathbf{r},L)$	Irradiance of beam in free space
$I(\mathbf{r},L)$	Irradiance of beam in random medium
$I_\nu(x)$	Modified Bessel function of order ν
i_{IF}	Intermediate frequency (IF) signal current
i_S	Signal current in a detector
i_N	Shot noise current in a detector
$J_\nu(x)$	Bessel function of order ν
k	Wave number of beam wave (= $2\pi/\lambda$)
$K_\nu(x)$	Modified Bessel function of order ν
l_0	Inner scale of turbulence
L_0	Outer scale of turbulence
L	Propagation path length
MCF	Mutual coherence function
MTF	Modulation transfer function
$n(\mathbf{R})$	Index of refraction
$n_1(\mathbf{R})$, $n_1(\mathbf{r},z)$	Random fluctuation in index of refraction
OTF	Optical transfer function
PSF	Point spread function
P_d	Probability of detection
P_{fa}	Probability of false alarm
P_S	Signal power
$\mathbf{p}$	Transverse vector between two observation points
Q_l	Nondimensional inner-scale parameter (=$L\kappa_l^2/k$)

Q_0	Nondimensional outer-scale parameter ($=L\kappa_0^2/k$)
$\mathbf{r}$	Transverse position of observation point
r_0	Atmospheric coherence length
$\mathbf{R}$	Position vector in three dimensions
$S(\mathbf{r},L)$	Random phase
$S_I(\omega)$	Power spectral density of irradiance
SNR	Signal-to-noise ratio
SR	Strehl ratio
$U_0(\mathbf{r},z)$	Complex amplitude of the field in free space
$U(\mathbf{r},z)$	Complex amplitude of the field in random medium
W_0	Beam radius at transmitter
W	Beam radius in free space at receiver
W_B	Beam radius in free space at the waist
W_e	Effective beam radius in random medium at receiver
W_G	Radius of Gaussian lens
W_R	Radius of target (reflector) surface
WSF	Wave structure function
$\alpha(L)$	Extinction coefficient
α,β	Parameters of the gamma-gamma distribution
β_0^2	Rytov variance for a spherical wave
η_x, η_y	Nondimensional cutoff frequencies for filter functions
γ	Propagation path amplitude parameter
$\Gamma(x)$	Gamma function
$\Gamma_2(\mathbf{r}_1,\mathbf{r}_2,L)$	Mutual coherence function
$\Gamma_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4,L)$	Fourth-order moment of the field
$\delta(x-a)$, $\delta(\mathbf{K}-\mathbf{K}')$	Dirac delta function
θ_0	Isoplanatic angle
Θ_0	Beam curvature parameter at transmitter

Θ	Beam curvature parameter of the beam at receiver
Θ_e, Λ_e	Effective beam parameters at receiver
κ	Scalar spatial wave number
κ_l	Inner-scale wave number parameter (= $3.3/l_0$)
κ_m	Inner-scale wave number parameter (= $5.92/l_0$)
κ_x, κ_y	Large-scale and small-scale cutoff frequencies for filter functions
λ	Wavelength
Λ_0	Fresnel ratio of beam at transmitter
Λ	Fresnel ratio of beam at receiver
ν_0	Quasi-frequency associated with the irradiance covariance function
ρ	Scalar separation between two observation points
ρ_0	Transverse spatial coherence radius
σ_1^2	Rytov variance for a plane wave
σ_B^2	Rytov variance for a Gaussian-beam wave
σ_G^2	Rytov variance for a Gaussian-beam wave with inner scale
σ_i^2, σ_e^2	Scintillation associated with the incident and echo waves in double-pass propagation
σ_{ie}^2	Scintillation arising from correlations between incident and echo waves
σ_I^2	Scintillation index (normalized irradiance variance)
$\sigma_I^2(D)$	Irradiance flux variance for a collecting aperture of diameter D
$\sigma_{I,l}^2$	Longitudinal component of scintillation index
$\sigma_{I,r}^2$	Radial component of scintillation index
σ_N^2	Total noise power in detector current
σ_P^2	Rytov variance for a plane wave with inner scale
σ_S^2	Rytov variance for a spherical wave with inner scale

$\sigma_x^2,\ \sigma_y^2$	Large-scale and small-scale scintillations
$\sigma_{\ln I}^2,\ \sigma_{\ln I,r}^2,\ \sigma_{\ln I,l}^2$	Log-irradiance variances
$\sigma_{\ln x}^2,\ \sigma_{\ln y}^2$	Large-scale and small-scale log-irradiance variances
σ_χ^2	Log-amplitude variance
τ	Transmittance
$\Phi_n(\kappa)$	Three-dimensional spatial power spectrum of refractive index
$\chi(\mathbf{r},L)$	Random log-amplitude
$\psi_1(\mathbf{r},L),\ \psi_2(\mathbf{r},L)$	Complex phase perturbations of Rytov approximation
$\psi_1(\mathbf{r},\mathbf{s}),\ \psi_2(\mathbf{r},\mathbf{s})$	Complex phase perturbations of extended Huygens-Fresnel principle
Ω_G	Fresnel ratio characterizing radius of Gaussian lens
Ω_R	Fresnel ratio characterizing radius of reflector
$< >$	Ensemble average