Show that

$$\frac{B_\chi(\rho)}{B_\chi(0)} = {}_1F_1\left(\frac{7}{6};1;-\frac{\kappa_m^2\rho^2}{4}\right).$$

14. Starting with Eq. (133), derive Eq. (134) by expressing the complex expressions in polar coordinates, i.e., write

$$x + iy = re^{i\theta},$$

where $r = \sqrt{x^2+y^2}$ and $\theta = \tan^{-1}(y/x)$. Consequently, verify Eqs. (135).

15. By directly evaluating the integrals, verify Eq. (145).

16. Verify that the substitution of (154) into (153) leads to (156).

17. Given a Gaussian-beam wave at the transmitter with beam characteristics $W_0 = 0.03\,\text{m}$, $F_0 = 500\,\text{m}$, $\lambda = 0.633\,\mu\text{m}$, determine W, F and the on-axis intensity $I(0,L)$ at distance L = 1200 m from the transmitter. Assume unit amplitude at the transmitter.

Ans. $W = 0.043\,\text{m}$, $F = -710.5\,\text{m}$, $I(0,L) = 0.492\,\text{W/m}^2$

18. If the beam described in Problem 17 passes through a lens/aperture stop of radius 0.01 m and focal length 0.05 m at distance 1200 m from the transmitter, what is the spot radius, phase front radius of curvature, and mean (on-axis) intensity of the beam at distance 0.1 m behind the lens?

Ans. $W = 9.5\,\text{mm}$, $F = -5\,\text{cm}$, $I(0,L) = 0.993\,\text{W/m}^2$

19. By expressing the radial polynomials (166) of the Zernike set in terms of Pochhammer symbols,

(*a*) show that

$$R_n^m(r) = n!\left(\frac{n-m}{2}\right)!\left(\frac{n+m}{2}\right)!$$
$$\times\ {}_2F_1\left(-\frac{n-m}{2},-\frac{n+m}{2};-n;\frac{1}{r^2}\right).$$

(*b*) From part (*a*), verify that

$$R_n^m(1) = 1.$$

20. Follow the technique in Section 15.8.3, used for evaluating $G_{2,\text{even}}(\kappa,\varphi)$ (corresponding to $Z_2[1,1]$), to deduce that (corresponding to $Z_3[1,1]$)

$$(a)\ G_{3,\text{odd}}(\kappa,\varphi) = 4i\frac{2J_2(\kappa)}{\kappa}\sin\varphi.$$

(*b*) Corresponding to the Zernike function $Z_4[0,2]$, calculate the filter function $G_{4,\text{even}}(\kappa,\varphi)$.