

**Use of Optical
Correlation Techniques
for Characterizing Scattering Objects and Media**

**Oleg V. Angelsky
Steen G. Hanson
Peter P. Maksimyak**

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PREFACE

Optical methods are known to provide some of the most important information obtained in studying the surrounding world. The amplitude, phase, polarization, spectral, angular, and correlation characteristics of optical fields may carry vital information on objects of interest. Modern fast-acting optical processing systems are capable of collecting information on diagnostically important parameters with high efficiency. This is achieved primarily by parallel processing large amounts of data and by taking advantage of analog optical computing devices. The problem of metrology of random optical fields, such as speckle fields resulting from the interaction of laser beams with a random phase object, has a significant place in optical diagnostics and optical recognition studies. These problems range from astronomy to industrial quality control.

In the present monograph, consideration is given to interference, polarization interference, and holographic correlometry of amplitude and phase parameters of random optical fields for the purpose of subsequent diagnostics of the appropriate objects and media. An attempt is made to show the possibilities for diagnostics of light-scattering objects and media provided by utilization of the properties of coherent optical radiation. The diagnostics of rough surfaces, being of great importance for many branches of industry, is particularly emphasized throughout the monograph.

An important feature of this monograph is the adaptation of the ideas formulated in classic work on statistical radiophysics and optics to diagnostic applications. At the same time, the proposed diagnostic techniques are aimed at obtaining the maximum information on objects, such as the roughness height (or phase increment) distribution functions for random phase objects, size distribution functions for dispersive systems, or root-mean-square velocity distribution functions for Brownian particles. Some of the diagnostic methods considered are unconventional. The proposed group of unique techniques for interference measurement of statistical moments of the field amplitude and phase up to the third and fourth orders constitutes a particular class of statistical methods for interference correlometry of optical fields. Most of the methods discussed result from physical modeling.

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Oleg V. Angelsky
Steen G. Hanson
Peter P. Maksimyak
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INTRODUCTION

The discussion here will be limited to linear and classical wave problems. Using the classification of statistical wave problems, the subject matter of this monograph can be defined as follows.

Let the propagation of a light wave $u(r,t)$ be described by any linear spatial-temporal operator L , such that the wave field satisfies the equation $Lu(r,t)=q(r,t)$, where $q(r,t)$ designates the field sources. According to the statistical wave theory, the following quantities can be random:

- (1) the field sources (real or virtual);
- (2) the properties of the medium through which the field propagates;
- (3) the shapes and the positions of the interfaces;
- (4) the conditions for field recording.

To obtain information on the scattering objects and the propagation medium, measurements of either the diffracted or the scattered fields are required. In this monograph, both the primary source (laser) and the recording conditions will be assumed to be deterministic. In other words, we consider here the inverse diagnostic problems corresponding to items (2) and (3) of the above classification.

The central idea of the book is to treat thoroughly the interrelation between the statistical and deterministic parameters of the object under study and those of the resulting optical field. Interferometry, polarization interferometry, optical velocimetry, and holography of random fields are considered in the context of this main idea, and an attempt is made to use the above techniques for obtaining experimental data that would be sufficient for a complete description of random optical fields.

The objects to be studied are of different natures. We consider random phase objects that transform the spatial phase distribution of the probing beam, causing both amplitude and phase modulations or even transformation of the longitudinal correlation function of the optical field. The other objects, such as complex objects modulating amplitude and phase of the field, turbulent liquids or particles undergoing Brownian motion, will cause temporal transformations of the amplitude and the phase of the field.

Various spatial-temporal analogies concerning random fields are used throughout this book. In particular, the theory of stochastic oscillations and its applicability in dealing with spatial chaos in stationary random fields is established.

Chapter 1 gives the outlines of the theory of probability and mathematical statistics as applied to random non-uniform media and fields. The models that

describe scattering of optical radiation by continuous, randomly non-uniform media and rough surfaces and scattering by dispersive media are given.

Several modern methods (which developed from quite traditional ones) for measuring the statistical parameters of interface roughness are reviewed in Chapter 2. This chapter is intended only to give an idea of the basic knowledge on the problem of rough surface metrology that has been developed through new approaches and techniques (called optical correlation methods) that are discussed in the following chapters.

The angle-resolved scattering (ARS) method, total integrated scattering (TIS), and the bidirectional reflectance distribution function (BRDF), as well as the heterodyning method for roughness diagnostics are discussed here. Impressive sensitivity down to several angstroms has been realized when these methods are used for estimation of the height parameters of rough surfaces. Two-dimensional topological mapping of rough surfaces and establishing a means for direct roughness measurements in the framework of the angle-resolved scattering method seem to be the most promising directions for further development of the approaches discussed.

What are the reasons for combining these well-known methods in one group and considering them in separate a chapter? First of all, these methods are based on models of light scattering by rough surfaces using Kirchhoff's approximation or the small perturbation approximation. Second, the roughness metrology here is based on the data contained in angular (spatial) and temporal intensity distributions, rather than in such distributions of the field's amplitude and phase, so that only the photometric quantities are used as the parameters for the field diagnostics. Finally, the above methods were the first to be implemented in industrial diagnostic systems. The distinguishing features mentioned permit us to separate the methods of optical correlation metrology considered in Chapter 3 from the ones discussed in Chapter 2.

Direct interference methods for measuring statistical parameters of inhomogeneous phase objects are considered primarily in Chapter 3. These methods are based on the use of reciprocal interconnections between the statistical parameters of an object and the mixed amplitude and phase statistical moments of the scattered field. Slightly rough surfaces, turbulent liquids, and optical crystals with phase variances less than unity and correlation lengths exceeding considerably the wavelength of the probing beam are examples of the objects under study in this chapter. Calculations are performed within the framework of a model of a random phase object that is described in the opening part of this chapter.

Roughness diagnostic techniques based on the measurement of the relative visibility (scintillation index) of a speckle field resulting from the interaction of a coherent probing beam with the object's surface are first presented. Heterodyning or light-beating methods for roughness diagnostics are based on measuring the transformation of the spatial-temporal content of the radiation scattered from moving objects. This technique has been employed in many industrial devices for roughness metrology, with resulting sensitivities in the angstrom range.

Furthermore, roughness measuring techniques are treated based on the measurement of statistical parameters of an object field, such as the transverse coherence function, the amplitude dispersion, the phase variance, and the mixed amplitude and phase statistical moments of the field up to the third and fourth order. A possibility for the measurement of the height distribution function interferometrically will be shown. The fundamental advantages of the techniques presented, such as high sensitivity (down to several angstroms) and fast-acting (down to 1 sec) are emphasized.

The authors believe that the following important questions will be answered in Chapter 3:

- What is a method for comprehensive statistical characterization of a slightly rough surface that can be obtained by applying an optical metrology?
- What are the means for overcoming limitations and difficulties intrinsic to interferometry of roughness?
- What are the means for increasing operating speed and sensitivity for measurement of slightly rough surfaces?
- How would the roughness of spherically or cylindrically shaped surfaces be measured optically?

The methods discussed are promising for industrial quality control of surface finish, including optical, electronic, and fine mechanical systems. Examples of the application of the interference techniques to control silicon and germanium substrates for microelectronic circuits are presented as well. Such control can be carried out at various stages of the production process, including the stage following dusting off of the substrates. If line production of such substrates is needed, these fast-acting techniques are especially attractive, as well as in production control of the optical surfaces of plane, spherical, or cylindrical forms. Examples of fast-acting quality control of metallic mirrors during finishing microsharpening using diamond dust are also given. In the optical industry, integral (area-covering) fast-acting surface control of thin flat plates is also important. This problem is considered, and the appropriate version of the above interference technique is developed for this particular case. Furthermore, quality control of cylindrical surfaces of the rollers used in the photochemical, cellulose, and paper-converting industries is treated.

Finally, possibilities for increasing interference profilometry accuracy by perfecting the statistical algorithms for data processing are analyzed.

Chapter 4 is devoted to the problem of diagnostics of surfaces with large inhomogeneities by using a reciprocal interconnection of statistical parameters of the object's roughness and the corresponding statistical amplitude and phase parameters of the field. The essence of the problem exists in the loss of any unambiguous interrelationship between amplitude and phase parameters of the field, on the one hand, and the corresponding structural parameters of an object on the other.

One version of the solution to this problem is proposed, which deviates from the framework of traditional approaches. Concepts of chaos theory and fractal geometry are introduced for this purpose. It is presumed that both the object of

interest and the scattered field can be characterized by using chaos dimensionality and then by looking for a diagnostically important interrelationship between them. Recent studies devoted to this topic are analyzed. New algorithms for optical metrology of chaos are proposed as well as appropriate experimental arrangements. The problem of diagnostics of low-dimensional signals wrapped in high-dimensional noise is considered. Although the level of consideration is academic to a certain extent, this problem has important applied significance for novel diagnostic tasks and the means for solving them.

The following questions, which are the subject of a long-term discussion in the technical literature, will be answered from an unconventional viewpoint in Chapter 4:

- Is the problem of optical interferometry of a rough surface with a phase variance exceeding unity (when the phase difference of a neighboring pixel is more than 2π ...) solved?
- Do fractal optics concepts provide promising means and prospects for solving the problem of rough surface diagnostics?
- In what manner do the conventional approach and the novel one based on the fractal optics concept relate to each other?

Most likely, the proposed answers will be the subject of further discussion. If this is so, our purpose will be achieved.

Chapter 5 deals with optical correlation diagnostics of structural and dynamical parameters of light-scattering media based on the reciprocal interconnection between the correlation functions of the probing beam and the scattered beam.

We hope that the reader will find here the answers to the following questions:

- What are the advantages of an optical correlation approach to diagnostics of a light-scattering particle ensemble (dispersive media)?
- What are the advantages possessed by the continuous-exposure holographic technique applied to the study of an ensemble of Brownian light-scattering particles?

A possibility for measuring the function of the partial signal retardations is shown, and estimates of the complex refractive index of the medium are given. The height distribution function for surfaces with large inhomogeneities can be reconstructed using the data obtained from the transformed longitudinal correlation function of the field.

Holographic techniques for reconstruction of the velocity distribution function for Brownian particles and for particle sizing are represented. These methods are based on measuring the temporal correlation function of the field scattered by the dynamic system under investigation. Possibilities for high-accuracy metrology of Doppler shifts are realized using this technique, permitting a temporal frequency resolution of the Doppler-shifted signals down to 10^{-5} – 10^{-6} Hz to be achieved.

The possibilities for sizing of spherical and non-spherical particles by measuring the transverse coherence function of the radiation field in the image plane of a monolayer of such particles are also discussed in Chapter 4. A data-processing algorithm for the diagnostic system is given that can be implemented electronically. Possibilities for metrological systems to be developed based on this approach are analyzed.

The significance of applying the proposed metrological systems is discussed. In particular, knowledge of the Brownian motion parameters of unicellular algae enables the observer to predict their proliferation in an aqueous medium, which is an important hydrobiological problem. Furthermore, applications of this technique for rapid and high-fidelity sizing of light-scattering particles in a dispersive medium range from powder metallurgy to the life sciences. In particular, the study of aggregating blood elements using the technique discussed has just been started.