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Abstract. The amount of echo data is very large in highly squinted synthetic aperture radar (SAR) imaging with high resolution. To solve this problem, an imaging method for highly squinted SAR with under-sampled echo data based on compressed sensing (CS) is put forward. First, the echo signal model of highly squinted SAR is analyzed and a nonlinear chirp scaling (NCS) imaging method with the Nyquist-sampled echo data is proposed, with which the range walk is corrected and the range-azimuth coupling is mitigated. Based on the imaging method, the NCS operator is established. Combining the NCS operator and CS theory, a highly squinted SAR imaging scheme is formulated. The modified iterative thresholding algorithm is utilized to solve the imaging scheme, which forms a highly squinted SAR imaging method. With the proposed method, just a small amount of imaging data is required for highly squinted SAR imaging. Finally, the effectiveness of the proposed method is proven by the simulations. © *The Authors. Published by SPIE under a Creative Commons Attribution 3.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI.* [DOI: 10.1117/1.JRS.9.097495]

Keywords: synthetic aperture radar; highly squinted imaging; compressed sensing; nonlinear chirp scaling operator; iterative thresholding algorithm.

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1 Introduction

Synthetic aperture radar (SAR) can realize imaging for ground targets all day and under all weather conditions, which can achieve a high resolution in the range direction due to the use of high transmitted-pulse bandwidth and in the azimuth direction due to the storage of data over a certain observation time.^{1,2} In the conventional SAR imaging mode, the pointing direction of the antenna is nearly perpendicular to the flight path. However, on some occasions, in order to point at an angle from the broadside, squint mode is adopted to increase the flexibility of SAR.^{3,4} Compared to the broadside SAR, squint SAR can give repeat observations on the same region during its whole flight track.⁵ With this prominent advantage, the highly squinted SAR is based on the matched filter (MF) algorithm, which needs Nyquist samples of the echoes. However, along with the improvement of range resolution, the SAR imaging system requires increasing measurements, storage, and downlink bandwidth.⁷ Hence, we intend to find an imaging method for highly squinted SAR, which can not only guarantee the quality of the imaging results but also dramatically reduce the amount of required echo data.

In the last few years, the compressed sensing (CS) theory was introduced in Refs. 8 and 9, which indicates that one can stably and accurately reconstruct nearly sparse signals from dramatically under-sampled data in an incoherent domain. With this prominent advantage, the CS theory has been used in SAR data processing to reduce the data amount and enhance the system

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performance.¹⁰⁻¹⁷ A compressive radar imaging scheme based on the CS theory was first reported by Baraniuk and Steeghs,¹¹ with which the pulse compression MF is no longer needed. In Ref. 12, the SAR raw data are compressed and the imaging result is reconstructed by using the CS framework with real wavelets. References 13 and 14 convert the two-dimensional (2-D) SAR imaging problem into a subsignal collection problem, which could dramatically reduce the amount of raw data. However, high computation and memory costs are required to decompose and recover the 2-D subsignals. The CS processing is performed after the conventional range compression in Refs. 15 and 16, which could acquire a high-resolution azimuth profile. In Ref. 17, a 2-D sparse SAR imaging scheme with stepped-frequency waveform is put forward. The advantage of this method is that only a small number of frequencies and echo data are needed to reconstruct the image of targets. However, all of these works are suitable only for broadside SAR. There are few reference works devoted specifically to highly squinted SAR imaging combining with CS theory. Due to the special imaging geometry, the highly squinted SAR brings new complexities and challenges to the imaging process. Thus, the aforementioned CS-SAR imaging methods cannot be utilized in highly squinted SAR imaging with the undersampled echo data.

In this paper, an imaging method for highly squinted SAR based on the CS theory is put forward. First, an imaging method for highly squinted SAR with the Nyquist-sampled data is proposed. The range walk removal function is constructed to correct the linear range walk and the nonlinear chirp scaling (NCS) method is applied to mitigate the range-azimuth coupling. Second, in order to achieve the highly squinted SAR imaging with the under-sampled echo data, the NCS operator and CS imaging scheme are formulated based on the aforementioned imaging method. And then, the imaging result is obtained by using the modified iterative thresholding algorithm (ITA). The main contributions of the present work can be concluded as follows:

- 1. In order to facilitate the analysis of imaging method with the under-sampled echo data, a new NCS imaging method with Nyquist-sampled echo data is put forward, with which the range walk can be corrected and the range-azimuth coupling can be mitigated.
- 2. The NCS operator and CS imaging scheme based on the Nyquist-sampled imaging method are established. The modified ITA is utilized to solve the imaging scheme, which forms a highly squinted SAR imaging method. By using the method, just a small amount of imaging data is required for highly squinted SAR imaging.

The rest of the paper is organized as follows. Section 2 gives the signal and geometry model of the highly squinted SAR. An NCS imaging method based on range walk removal is also introduced in this section. In Sec. 3, the NCS operator and CS imaging scheme are constructed. Moreover, the modified ITA is utilized to solve the imaging scheme. Simulation results are presented in Sec. 4 to validate the effectiveness of proposed approach. Finally, we make some conclusions in Sec. 5.

2 Imaging Algorithm for Highly Squinted Synthetic Aperture Radar with Nyquist-Sampled Data

There are some papers that have put forward highly squinted SAR imaging methods with Nyquist-sampled data.^{3,4} However, it is difficult to obtain the highly squinted imaging operator from these methods. The imaging operator is the key factor in CS SAR imaging method. In order to facilitate the construction of the highly squinted imaging operator, we first focus on establishment of an imaging approach for highly squinted SAR with Nyquist-sampled data in this section.

2.1 Echo Signal Model for Highly Squinted Synthetic Aperture Radar

The squinted SAR geometry is shown in Fig. 1. An SAR sensor travels along a straight-line flight path during a synthetic aperture length L. The velocity and height of platform are v and h, respectively. During the data acquisition, the radar beam directs at the target with a squinted angle θ_s . Symbol R_0 denotes the vertical distance between the scene center and the flight path.



Fig. 1 Imaging model of highly squinted synthetic aperture radar (SAR): (a) flow chart of highly squinted SAR; (b) geometric model of flight path and point scatterer.

The geometric model of flight path and point scatterer is shown in Fig. 1(b). The distance between the point scatterer P and the flight path is R_p . During the data acquisition, the transmitted chirp signal is

$$s(\hat{t}, t_m) = \operatorname{rect}\left(\frac{\hat{t}}{T_p}\right) \exp[j2\pi f_0 t + j\pi\gamma \hat{t}^2], \qquad (1)$$

where rect $(\hat{t}/T_p) = 1$ is the rectangular window with a width of T_p , T_p is the pulse repetition interval. Term \hat{t} denotes range fast time, $t = \hat{t} + t_m$, and t_m is the azimuth slow time. Term f_0 is the carrier frequency, and γ is the chirp rate. After downconversion to the baseband, the received signal from the point scatterer P is given by

$$s_r(\hat{t}, t_m; R_p) = \operatorname{rect}\left[\frac{\hat{t} - 2R(t_m; R_p)/c}{T_p}\right] \exp\left\{j\pi\gamma \left[\hat{t} - \frac{2R(t_m; R_p)}{c}\right]^2\right\} \exp\left[-j\frac{4\pi R(t_m; R_p)}{\lambda}\right], \quad (2)$$

where $R(t_m; R_p)$ is the instantaneous range from the point scatterer P to radar, and it can be expressed as

$$R(t_m; R_p) = \sqrt{R_p^2 + (vt_m - X_p)^2 - 2R_p(vt_m - X_p)\sin\theta_s}.$$
(3)

Expanding the distance $R(t_m; R_p)$ in Taylor's series and neglecting the higher-order terms, $R(t_m; R_p)$ can then be expressed approximately as follows:

$$R(t_m; R_p) = R_p - (vt_m - X_p) \sin \theta_s + \frac{(vt_m - X_p)^2 \cos^2 \theta_s}{2R_p}$$

= $\sqrt{R_p^2 + \cos^2 \theta_s (vt_m - X_p)^2} - (vt_m - X_p) \sin \theta_s.$ (4)

2.2 Nonlinear Chirp Scaling Imaging Method Based on Range Walk Removal

Applying Fourier transform (FT) with respect to \hat{t} , the signal $s_r(\hat{t}, t_m; R_p)$ is transferred into range Doppler domain (i.e., range time and azimuth frequency domain), it yields

$$S_r(f_r, t_m; R_p) = \exp\left(-j\pi \frac{f_r^2}{\gamma}\right) \exp\left[-j\frac{4\pi R(t_m; R_p)(f_r + f_0)}{c}\right].$$
(5)

Submitting Eq. (4) into Eq. (5), we can obtain

$$S_r(f_r, t_m; R_p) = \exp\left[-j\frac{4\pi X_p \sin\theta_s(f_r + f_0)}{c}\right] \exp\left[j\frac{4\pi v t_m \sin\theta_s(f_r + f_0)}{c}\right] \\ \times \exp\left(-j\pi\frac{f_r^2}{\gamma}\right) \exp\left[-j\frac{4\pi\sqrt{R_p^2 + \cos^2\theta_s(v t_m - X_p)}(f_r + f_0)}{c}\right].$$
(6)

As can be seen from Eq. (6), the second exponential term of $S_r(f_r, t_m; R_p)$ is the linear range walk term. Therefore, the range walk removal function $H_1(f_r, t_m)$ can be written as

$$H_1(f_r, t_m) = \exp\left[-j\frac{4\pi v t_m \sin\theta_s(f_r + f_0)}{c}\right].$$
(7)

To obtain the 2-D spectrum, we apply FT in azimuth to the aforementioned compensated signal, i.e.,

$$S(f_r, f_a; R_p) = \exp\left(-j\pi \frac{f_r^2}{\gamma}\right) \exp\left[-j\frac{4\pi(X_p \sin \theta_s)(f_r + f_0)}{c}\right] \\ \times \exp\left(-j2\pi f_a \frac{X_p}{v}\right) \exp[j\Xi(f_r, f_a; R_p)],$$
(8)

where $\Xi(f_r, f_a; R_p) = -(4\pi R_p f_0/c)\sqrt{(1 + f_r/f_a)^2 - (f_a\lambda/2v\cos\theta_s)^2}$. We can find that $\Xi(f_r, f_a; R_p)$ is the coupling term of the range frequency f_r and azimuth Doppler f_a . It has the characteristic of spatial variance. In order to obtain the focused imaging result, $\Xi(f_r, f_a; R_p)$ should be accurately compensated. Expanding it in Taylor's series, we can obtain

$$\Xi(f_r, f_a; R_p) = \phi_0(f_a) + \phi_1(f_a)f_r + \phi_2(f_a)f_r^2 + \phi_3(f_a)f_r^3 + o(f_a, f_r^4), \tag{9}$$

where

$$\begin{split} \phi_0(f_a) &= -\frac{4\pi R_p f_0}{c} B(f_a); \quad \phi_1(f_a) = -\frac{4\pi R_p}{cB(f_a)}; \quad \phi_2(f_a) = -\frac{2\pi R_p}{cf_c} \frac{1 - B^2(f_a)}{B^3(f_a)}; \\ \phi_3(f_a) &= \frac{2\pi R_p}{cf_c^2} \frac{1 - B^2(f_a)}{B^5(f_a)}; \quad B(f_a) = \sqrt{1 - \left(\frac{f_a\lambda}{2v\cos\theta_s}\right)^2}. \end{split}$$

The chirp rate $\gamma_e(f_a, R_p)$ of $S(f_r, f_a; R_p)$ is equivalent to the following expression:

$$\frac{1}{\gamma_e(f_a, R_p)} = \frac{1}{\gamma} + \frac{2R_p}{cf_c} \frac{1 - B^2(f_a)}{B^3(f_a)}.$$
(10)

Inspecting Eq. (10), $\gamma_e(f_a, R_p)$ is dependent on the Doppler frequency f_a and the distance R_p , which means the signal $S(f_r, f_a; R_p)$ has the characteristic of spatial variance. However, the spatial variance is not considered in the conventional chirp scaling algorithm (CSA). Thus, if using the conventional CSA, the imaging edge would be defocused. The NCS algorithm has considered the spatial variance. Therefore, we adopt it for the following imaging processing.

Before the NCS processing, the third-order term of $S(f_r, f_a; R_p)$ should be filtered. According to the above analysis, the filtered function $H_2(f_r)$ is given by

$$H_2(f_r) = \exp[-j\phi_3(f_a)f_r^3].$$
(11)

Multiplying Eq. (11) with Eq. (8) and transforming the result into the range Doppler domain by the range inverse FT (IFT), we can obtain

$$s'(\hat{t}, f_a; R_p) = \exp\left(-j\frac{4\pi X_p \sin\theta_s f_0}{c}\right) \exp\left(-j2\pi f_a \frac{X_p}{v}\right) \exp\left[-j\frac{4\pi R_p}{\lambda} B(f_a)\right] \\ \times \exp\left\{-j\pi\gamma_e(f_a, R_p)\left[\hat{t} - \frac{2X_p \sin\theta_s}{c} - \frac{2R_p}{c}\frac{1}{B(f_a)}\right]^2\right\}.$$
 (12)

According to the NCS algorithm, the NCS operation function can be written as

$$H_3(\hat{t}, f_a; R_0) = \exp\{j\pi q_2(f_a)[\hat{t} - \tau_d(f_a, R_0)]^2\},\tag{13}$$

where $q_2(f_a)$ is the modulation rate of $H_3(\hat{t}, f_a; R_0)$, which is abbreviated as q_2 . Multiplying Eq. (13) with Eq. (12) and applying the principle of stationary phase to the result, we can obtain the result in the 2-D frequency domain as

$$S'(f_r, f_a; R_p) = \exp\left(-j\frac{4\pi X_p \sin \theta_s f_0}{c}\right) \exp\left(-j2\pi f_a \frac{X_p}{v}\right) \\ \times \exp\left[-j\frac{4\pi R_0}{\lambda}B(f_a)\right] \exp\left[-jZ(f_r, f_a, \Delta\tau)\right], \tag{14}$$

where $\Delta \tau = \tau_d(f_a, R_0) - \tau_d(f_a, R_p)$. There are four exponential terms in Eq. (14). The first exponential term determines the range displacement, which is dependent on target azimuth position and squint angle; the second term determines the azimuth position of the target; the third term is the azimuth-independent phase modulation term; the fourth exponential term is the cross-coupling term. Expanding the fourth exponential term into a power series of f_r , we can obtain

$$Z(f_r, f_a, \Delta \tau) = C(q_2, f_a, f_r, f_r^2) + D(q_2, f_a) \Delta \tau f_r.$$
(15)

The first term of Eq. (15) is independent from the range distance, and the second term is the main factor that causes linear shift in the range direction. In order to eliminate the geometric distortion induced by the spatial variance, we set

$$D(q_2, f_a) = \frac{1}{\alpha(f_a)}, \quad \alpha(f_a) = \frac{1}{B(f_a)}.$$
(16)

Solving Eq. (16), the modulation rate q_2 can be written as

$$q_2 = \gamma_e(f_a, R_p)[\alpha(f_a) - 1].$$
(17)

Submitting Eq. (17) into Eq. (14), we can obtain

$$S'(f_r, f_a; R_p) = \exp\left(-j\frac{4\pi X_p \sin \theta_s f_0}{c}\right) \exp\left(-j2\pi f_a \frac{X_p}{v}\right) \exp\left[-j\frac{4\pi R_0}{\lambda} B(f_a)\right] \\ \times \exp\left[-j2\pi \tau_d'(f_a, R_p)f_r\right] \exp\left[-j\frac{\pi}{\alpha(f_a)\gamma_e(f_a, R_p)}f_r^2\right] \exp\left[jM(f_a, R_p)\right], \quad (18)$$

where

$$\begin{cases} M(f_a, R_p) = 4\pi\gamma_e(f_a, R_p)a(f_a)[a(f_a) - 1]\Delta\tau^2\\ \tau_d'(f_a, R_0) = \frac{2\{X_p \sin \theta_s + R_p + [\alpha(f_a) - 1]R_0\}}{c} \end{cases}$$

From Eq. (18), the range compression function and range cell migration correction (RCMC) function can be given by

$$H_4(f_r, f_a; R_p) = \exp\left\{j\frac{4\pi[a(f_a) - 1]R_0}{c}f_r\right\} \exp\left[j\frac{\pi f_r^2}{\alpha(f_a)\gamma_e(f_a; R_p)}\right].$$
 (19)

Multiplying Eq. (19) with Eq. (18), we can complete range compression and RCMC. In order to implement the azimuth processing, we transform the result into the range Doppler domain by range IFT. The azimuth compression and residual phase compensated functions $H_5(f_a, \Delta \tau; R_p)$ are written as

$$H_5(f_a, \Delta \tau; R_p) = \exp\left[j\frac{4\pi R_0}{\lambda}\mathbf{B}(f_a)\right] \exp[-jM(f_a, R_p)].$$
(20)

Then azimuth IFT is applied to the aforementioned result back into the slow-time domain. The imaging result of highly squinted SAR with the Nyquist samples is obtained. The focused imaging result is with some degree of geometric distortion in range and azimuth directions. The detailed geometric correction method is not discussed in this paper, which can be found in Ref. 4.

3 Imaging Method with Under-Sampled Data

The imaging method for highly squinted SAR with Nyquist-sampled data is proposed in Sec. 2.2. However, the amount of echo data is huge with high-resolution imaging. Therefore, in order to reduce the amount of echo data, the sampled ratio is set lower than that of the Nyquist requires. A sensing method for direct sampling and compressing analog signals is analog-to-information conversion (AIC).¹⁰ We adopt the AIC framework with random under-sampling scheme in range direction. The essence of this scheme is to nonuniformly undersample the echo signal. As for the under-sampled echo data, the aforementioned proposed method with Nyquist-sampled data becomes invalid. An imaging algorithm should be proposed.

The superiority of CS lies in that it merges sensing and compressing together, and a small number of "random" measurements can carry enough information to reconstruct the original signal. Thus, the CS theory is introduced to the highly squinted SAR imaging to reduce the amount of echo data. The CS imaging method is put forward with three sequenced steps: First, the NCS operator is constructed based on the imaging method proposed in Sec. 2.2. Second, CS imaging scheme based on the NCS operator is established in details. Third, the modified ITA is utilized to solve the imaging scheme. The relationship between the imaging method with Nyquist samples and with under samples is shown in Fig. 2.

3.1 Compressed Sensing Model for Highly Squinted Synthetic Aperture Radar

In order to facilitate the following analysis, the NCS imaging method based on range walk removal in Sec. 2.2 can be expressed as follows:

$$\Theta = \Gamma(s_r) = \mathbf{\omega}_a \cdot \{ \langle [(\{ [(\{\hat{\omega}_a \cdot [(s_r \cdot \mathbf{\omega}_r) \cdot H_1]\} \cdot H_2) \cdot \hat{\omega}_r] \cdot H_3\} \cdot \mathbf{\omega}_r) \cdot H_4] \cdot \hat{\omega}_r \rangle \cdot H_5 \},$$
(21)

where (\cdot) denotes the Hadamard product. Terms $\boldsymbol{\omega}$ and $\hat{\boldsymbol{\omega}}$, respectively, are the discrete FT (DFT) matrix and inverse DFT matrix (in practice, they are both implemented by FFT) to perform, the subscripts *a* and *r* denote the direction of azimuth and range along which the FFT performs. Term s_r is the echo signal matrix, the size of which is $N_a \times N_r$. Term N_a is the sampled number in azimuth direction and N_r is the number of range cells. Term $\boldsymbol{\Theta}$ is the imaging result for highly squinted SAR. Term $\Gamma(\cdot)$ is the NCS operator, which is the key factor in CS imaging model.

Equation (21) is a reversible process. If we input the complex image data, the echo signal can be obtained by implementing the inverse process of Eq. (21). The inverse process can be expressed as



Fig. 2 Relationship between imaging method with Nyquist samples and with under-samples.

$$s_r = \Gamma^{-1}(\Theta) = \{ \boldsymbol{\omega}_a \cdot \langle [(\{ [(\hat{\omega}_a \cdot \Theta) \cdot H_5^*] \cdot \boldsymbol{\omega}_r\} \cdot H_4^*) \cdot \hat{\omega}_r] \cdot H_3^* \} \cdot \boldsymbol{\omega}_r) \cdot H_2^*] \rangle \} \cdot H_1^* \cdot \hat{\omega}_r, \quad (22)$$

where $(\cdot)^*$ denotes conjugation. From the CS theory point of view, the under-sampled signal can be viewed as a low-dimensional measurement of Nyquist-sampled signal. The low-dimensional measurement matrix $\mathbf{\Phi} = \{\phi_{c,d}\}$ is a N_r -by- \hat{N}_r random partial unit matrix,¹⁸ where \hat{N}_r is the under-sampled number in range direction, and

$$\phi_{c,d} = \begin{cases} 1, & c = m_d \\ 0, & \text{others} \end{cases} \quad d = 1, \cdots, \hat{N}_r; \quad m_d \in 1, \cdots, N_r;.$$

$$(23)$$

The elements in each row vector of $\mathbf{\Phi}$ are 0, other than the m_d th element, where m_d is the random number. Thus, we can obtain

$$s_{rcom} = s_r \cdot \mathbf{\Phi}. \tag{24}$$

From Eq. (24), we can find that the size of under-sampled signal s_{rcom} is $N_a \times \hat{N}_r$. The undersampled ratio of echo signal is defined by $\eta = \hat{N}_r / N_r$. Submitting Eq. (24) into Eq. (22), we can obtain

$$s_{rcom} = \Gamma^{-1}(\Theta) \cdot \mathbf{\Phi}. \tag{25}$$

When $\Gamma^{-1}(\cdot) \cdot \Phi$ satisfies the restricted isometry property (RIP), the imaging result Θ can be obtained by solving the following optimization problem:

$$\min \|\Theta\|_0 \quad \text{subject to } s_{r\text{com}} = \Gamma^{-1}(\Theta) \cdot \mathbf{\Phi}, \tag{26}$$

where $\|\cdot\|_0$ denotes l_0 norm and min(·) denotes the minimization. That $\Gamma^{-1}(\cdot) \cdot \Phi$ satisfies the RIP is proven in detail in the Appendix.

3.2 Modified Iterative Thresholding Algorithm

How to solve the optimization problem of Eq. (26) is an important aspect in CS theory. Two majors are usually called " L_1 -minimization" and "greedy pursuit" and the others are the "nonconvex optimization" and the "Bayesian framework."¹⁹ Basis pursuit is one kind of the L_1 -minimization algorithm, which is based on the interior point.²⁰ ITA is another kind of the L_1 -minimization algorithm, which is known as a first-order algorithm.²¹ By the ITA, Eq. (26) can be transferred into the following expression:

$$\min_{\Theta} \left\{ \frac{1}{2} \| s_{rcom} - \Gamma^{-1}(\Theta) \cdot \Phi \|_2^2 + \lambda \| \Theta \|_1 \right\},$$
(27)

where λ is a regularization parameter, which is utilized to balance the precision and sparsity of the reconstructed result. Equation (27) can be effectively solved by ITA.²¹ The conventional ITA generates a sequence of approximates according to

$$\hat{\Theta}_{k} = \text{soft}[\hat{\Theta}_{k-1} + \mu A^{H}(s_{rcom} - A \cdot \hat{\Theta}_{k-1}), \lambda], \qquad (28)$$

where $\operatorname{soft}(x, \lambda) = \operatorname{sign}(x) \cdot \max(|x| - \lambda, 0)$, A is the measurement matrix, and $(\cdot)^{\mathrm{H}}$ denotes the conjugate transposition. Term μ controls the convergence of the ITA. However, A of Eq. (28) cannot be obtained. Thus, Eq. (28) should be modified to the following expression:

$$\hat{\Theta}_{k} = \operatorname{soft}\{\hat{\Theta}_{k-1} + \mu \cdot \Gamma[s_{rcom} - \Gamma^{-1}(\hat{\Theta}_{k-1}) \cdot \Phi] \Phi^{H}, \lambda\}.$$
(29)

There are two parameters λ and μ that need to be set. According to Refs. 7, 22, and 23, λ and μ can be set as

$$\mu_k = \frac{\|\Gamma^{-1}(\Delta\hat{\Theta}_k) \cdot \Phi\|_2^2}{\|\Delta\hat{\Theta}_k\|_2^2}, \qquad \lambda_k = \frac{|\hat{\Theta}_k + u_k \cdot \Delta\hat{\Theta}_k|_I}{u_k}, \tag{30}$$

 Table 1
 Synthetic aperture radar parameters.

Parameters	Values
Carrier frequency (f ₀)	10 GHz
Range bandwidth (B)	50 MHz
Platform velocity (v)	100 m/s
Pulse repetition interval (T_p)	10 <i>µ</i> s
Center slant range (R_B)	14.14 km
Squint angle (θ_s)	45 deg

where $|\cdot|_I$ is its I' th largest component in magnitude. The detailed steps of the modified ITA can be outlined as follows:

- Require: Under-sampled echo data s_{rcom} , NCS imaging operator $\Gamma(\cdot)$ and inverse imaging operator $\Gamma^{-1}(\cdot)$, low-dimensional measurement matrix Φ .
- Ensure: The imaging result Θ of highly squinted SAR.
- Step 1. Initialization: $\Theta_0 = 0$, the residual error $p_0 = s_{rcom}$, the maximum iteration K_{max} ;
- Step 2. For k = 0 to K_{max} do the following steps;
- Step 3. Matched filter on residual error: $\Delta \Theta_k = \Gamma(p_k \cdot \Phi^{\rm H});$

- Step 4. Compute the current estimation $\hat{\Theta}_{k+1}$: $\hat{\Theta}_{k+1} = \operatorname{soft}(\hat{\Theta}_k + u_k \cdot \Delta \hat{\Theta}_k, \lambda)$; Step 5. Update the residual error: $p_{k+1} = s_{rcom} \Gamma^{-1}(\hat{\Theta}_{k+1}) \cdot \Phi$; Step 6. Check the stopping criterion: if $|\hat{\Theta}_{k+1} \hat{\Theta}_k|_2 / |\hat{\Theta}_k|_2 > \rho$, then set k = k + 1 and go to Step 3; otherwise end.

Then $\hat{\Theta}(k)$ can be obtained, which is the imaging result for highly squinted SAR with undersampled echo data.

4 Experimental Analysis with Simulated and Measured Data

In this section, some simulations are conducted to demonstrate the effectiveness and feasibility of the proposed method. The experiments with simulated data using airborne SAR parameters shown in Table 1 are carried out.

The simulation uses an array of three targets, which are located in a 5 km \times 5 km grid in the slant range plane in azimuth/range, as shown in Fig. 3. Three targets have the same azimuth



Fig. 3 Flight geometry and target distribution in the slant plane for the simulation.

position and a distance of 2 km in the range direction. In the imaging methods, target P2 located at the center of observed scene is selected as the reference target.

First, the simulated echo data are sampled with the Nyquist theory. The proposed imaging algorithm in Sec. 2.2 is applied to process the simulated data. Figures 4(a) and 4(b) show the images of three targets when considering and not considering the spatial variance. It can be noted that the imaging result of target P2 is not affected by the spatial variance, which is focused well with both two methods. However, when not considering the spatial variance, the imaging results of targets P1 and P3 cannot be focused as shown in Fig. 4(b). By using the proposed method in Sec. 2.2, the targets P1 and P3 are focused quite well as shown in Fig. 4(a). Therefore, the proposed method in Sec. 2.2 is valid and feasible.

Next, in order to validate the proposed method in Sec. 3, the echo data are under-sampled by the random scheme. Let the under-sampled ratios η be 1/2 and 1/3. The reconstructed imaging results by using the CS imaging method are presented in Fig. 5. Figures 5(a) and 5(b) are the imaging results when η is 1/2 and 1/3, respectively. Comparing with Fig. 4(a), we can find that the sidelobes of Fig. 5 are lower and the quality of the resultant image is good enough. Therefore, by using the CS imaging method in Sec. 3, just a small amount of imaging data is required for highly squinted SAR imaging.

In addition, to further evaluate the performance of the proposed algorithm, the measured parameters peak sidelobe ratios (PSLRs) of target P2 in range and azimuth directions are calculated. PSLRs of target P2 in Fig. 4(a) are -13.21 and -13.01 dB, respectively, which are nearly the theoretical values. However, PSLRs of target P2 in Fig. 5(a) are -16.12 and -15.09 dB, respectively, which are much smaller than that of Fig. 4(a). It means that the quality of CS imaging method is better than that of imaging method proposed in Sec. 2.2.

In order to further validate our method, the simulated scene data are utilized in the following analysis. The simulation parameters are set the same as that of the above point target simulation. The observed scene is shown in Fig. 6. Figure 7(a) shows the imaging result without correcting geometric distortion by Nyquist-sampled imaging method, while Fig. 7(b) is the imaging result with geometric distortion corrected. Let the under-sampled ratio η be 3/4. The respective imaging results obtained by Nyquist-sampled imaging method and the proposed CS method are shown in Fig. 7(c) and 7(d). We can find that the Nyquist-sampled imaging method can only obtain image when samples are fully adopted. While η is 3/4,



Fig. 4 Imaging results under different situations: (a) imaging result with spatial variance considered; (b) imaging result with spatial variance not considered.

Gu et al.: Imaging method for highly squinted synthetic aperture radar...



Fig. 5 Results of compressed sensing (CS) imaging method with different under-sampled ratios: (a) imaging result with $\eta = 1/2$; (b) imaging result with $\eta = 1/3$.

the imaging quality is poor. However, the proposed CS method can perfectly recover the image, with a much reduced sidelobe. Figures 7(e) and 7(f) show the imaging results by the proposed CS imaging method with $\eta = 1/2$ and 1/4, respectively. When η is 1/2, the CS method can recover the image. However, when the under-sampled ratio is 1/4, the quality of the image is not very good; only the main information of observed scene can be obtained. It is easy to be comprehended and also according with the CS reconstructing theory. Therefore, the proposed method is valid and feasible.

Next, the peak signal-to-noise ratio (PSNR) and CPU times are utilized to compare the quality and efficiency of the proposed methods. The experiment is run in MATLAB 2010a on a computer with an Intel Pentium 3.2 GHz Dual-Core processor and 3GB memory. The comparison results are shown in Table 2. As can be observed from the Table, PSNR of $\eta = 3/4$ is higher than that of $\eta = 1/2$. It means that the quality of $\eta = 3/4$ is better, which is accordingly with the imaging results shown in Figs. 7(d) and 7(e). The CPU time comparison result shows that the Nyquist-sampled method is faster than the CS imaging method. However, the CPU time of the two methods is at the same level of magnitude.



Fig. 6 Observed scene.



Fig. 7 Imaging results of area targets with different methods and different under-sampled ratios: (a) imaging result of Nyquist samples with geometric distortion not corrected; (b) imaging result of Nyquist samples with geometric distortion corrected; (c) imaging result by Nyquist samples imaging method with $\eta = 1/2$; (d-f) the imaging results by the proposed compressed sensing method with $\eta = 3/4$, 1/2, and 1/4, respectively.

	Methods		
Indicators	Compressed sensing imaging method		Nyquist-sampled imaging method
Under-sampled ratio	3/4	1/2	
PSNR (dB)	45.55	40.84	
CPU time (s)	17.3	16.1	3.1

Table 2	Comparison of	peak signal-to-noise ratio	(PSNR) and CPU time.
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5 Conclusions

In this paper, an imaging method for highly squinted SAR with under-sampled echo data based on CS theory is proposed, which can solve the problem of the huge data amount in highly squinted SAR system. At first, the imaging method with the Nyquist-sampled echo data is proposed. Based on the method, the CS imaging method with the under-sampled echo data is then put forward. By using the proposed method, the highly squinted SAR imaging result can be obtained with a small amount of echo data.

However, the sampling rate of the proposed method is mainly determined by the sparsity of observed scene, which cannot be strictly defined. Even in some specific conditions, there are still many small targets in the background. In practice, we assume them to be zeros. As a result, the neglected small scatterers may affect the understanding of the imaging result. In order to obtain the small scatterers, the sampling rate of the proposed method cannot be reduced, which should be 100% Nyquist sampling rate. How to use our method for the nonsparse scene with undersampled echo data is our current research.

Appendix: Proof for the RIP of Matrix

From Eq. (22), $\Gamma^{-1}(\Theta)$ can be explicitly expressed by

$$\Gamma^{-1}(\Theta) = \{ \boldsymbol{\omega}_a \cdot \langle [(\{ [(\boldsymbol{\omega}_a' \cdot \Theta) \cdot H_5^*] \cdot \boldsymbol{\omega}_r\} \cdot H_4^*) \cdot \boldsymbol{\omega}_r'] \cdot H_3^*\} \cdot \boldsymbol{\omega}_r) \cdot H_2^*] \rangle \} \cdot H_1^* \cdot \boldsymbol{\omega}_r'.$$
(31)

Let θ denotes the vector form of Θ , namely, $\theta = \text{vec}(\Theta)$. According to Ref. 7, $\Gamma^{-1}(\Theta)$ can be written as matrices, and we have

$$\operatorname{vec}[\Gamma^{-1}(\mathbf{X})] = \mathbf{G} \cdot x = \hat{\omega}_r' \cdot \hat{H}_1^* \cdot \hat{\omega}_a \cdot \hat{H}_2^* \cdot \hat{\omega}_r \cdot \hat{H}_3^* \cdot \hat{\omega}_r' \cdot H_4^* \cdot \hat{\omega}_r \cdot \hat{H}_5^* \cdot \hat{\omega}_a' \cdot x,$$
(32)

where

$$\begin{split} \hat{\omega}_a &= I_{N_r} \otimes \mathbf{\omega}_a, \hat{\omega}_r = \mathbf{\omega}_r \otimes I_N \\ \hat{H}_1^* &= \text{diag}[\text{vec}(H_1^*)]; \\ \hat{H}_2^* &= \text{diag}[\text{vec}(H_2^*)]; \\ \hat{H}_3^* &= \text{diag}[\text{vec}(H_3^*)]; \\ \hat{H}_4^* &= \text{diag}[\text{vec}(H_4^*)]; \\ \hat{H}_5^* &= \text{diag}[\text{vec}(H_5^*)]. \end{split}$$

Thus, $s_{rcom} = \Gamma^{-1}(\Theta) \cdot \mathbf{\Phi}$ can be written as

$$S = \hat{\Phi} \cdot \mathbf{G} \cdot \boldsymbol{\theta},\tag{33}$$

where $S = \text{vec}(s_{rcom})$, $\hat{\Phi} = \Phi^{H} \otimes I_{n_{a}}$. To check whether $\Gamma^{-1}(\cdot) \cdot \Phi$ satisfies the RIP [Eq. (26)] means to check whether $\hat{\Phi} \cdot \mathbf{G}$ obeys the RIP. As discussed in Ref. 24, the randomly selected partial orthogonal matrix satisfies RIP. In this paper, the measurement matrix $\hat{\Phi}$ is a random partial unit matrix. Therefore, if the imaging operator \mathbf{G} is an orthogonal matrix, $\hat{\Phi} \cdot \mathbf{G}$ obeys the RIP.

The imaging operator **G** is an orthogonal matrix. $\hat{\omega}_a$ and $\hat{\omega}'_a$ are the DFT matrix and IDFT matrix, which are orthogonal matrices. Thus, we have

$$\hat{\omega}_a \cdot (\hat{\omega}_a)^H = \mathbf{I}\hat{\omega}'_a \cdot (\hat{\omega}'_a)^H = \mathbf{I}.$$
(34)

Meanwhile, we also can obtain

$$\hat{\omega}_r \cdot (\hat{\omega}_r)^H = \mathbf{I} \hat{\omega}_r' \cdot (\hat{\omega}_r')^H = \mathbf{I}.$$
(35)

 $\hat{H}_1^*, \hat{H}_2^*, \hat{H}_3^*, \hat{H}_4^*$, and \hat{H}_5^* are the diagonal plural matrices, respectively. Thus, we can obtain

$$\hat{H}_{1}^{*} \cdot (\hat{H}_{1}^{*})^{H} = \begin{bmatrix} \sigma_{1} & 0 & \cdots & 0 \\ 0 & \sigma_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{N_{r}N_{a}} \end{bmatrix},$$
$$\hat{H}_{2}^{*} \cdot (\hat{H}_{2}^{*})^{H} = \begin{bmatrix} \varepsilon_{1} & 0 & \cdots & 0 \\ 0 & \varepsilon_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \varepsilon_{N_{r}N_{a}} \end{bmatrix},$$
(36)

$$\hat{H}_{3}^{*} \cdot (\hat{H}_{3}^{*})^{H} = \begin{bmatrix} \zeta_{1} & 0 & \cdots & 0 \\ 0 & \zeta_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \zeta_{N_{r}N_{a}} \end{bmatrix},$$
$$\hat{H}_{4}^{*} \cdot (\hat{H}_{4}^{*})^{H} = \begin{bmatrix} \xi_{1} & 0 & \cdots & 0 \\ 0 & \xi_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi_{N_{r}N_{a}} \end{bmatrix},$$
(37)

$$\hat{H}_{5}^{*} \cdot (\hat{H}_{5}^{*})^{H} = \begin{bmatrix} \zeta_{1} & 0 & \cdots & 0 \\ 0 & \zeta_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \zeta_{N_{r}N_{a}} \end{bmatrix}.$$
(38)

Furthermore,

$$\mathbf{G} \cdot (\mathbf{G})^{\mathrm{H}} = \hat{\omega}_{r}' \cdot \hat{H}_{1}^{*} \cdot \hat{\omega}_{a} \cdot \hat{H}_{2}^{*} \cdot \hat{\omega}_{r} \cdot \hat{H}_{3}^{*} \cdot \hat{\omega}_{r}' \cdot H_{4}^{*} \cdot \hat{\omega}_{r} \cdot \hat{H}_{5}^{*} \cdot \hat{\omega}_{a}' \\
\cdot (\hat{\omega}_{r}' \cdot \hat{H}_{1}^{*} \cdot \hat{\omega}_{a} \cdot \hat{H}_{2}^{*} \cdot \hat{\omega}_{r} \cdot \hat{H}_{3}^{*} \cdot \hat{\omega}_{r}' \cdot H_{4}^{*} \cdot \hat{\omega}_{r} \cdot \hat{H}_{5}^{*} \cdot \hat{\omega}_{a}')^{\mathrm{H}} \\
= \hat{\omega}_{r}' \cdot \hat{H}_{1}^{*} \cdot \hat{\omega}_{a} \cdot \hat{H}_{2}^{*} \cdot \hat{\omega}_{r} \cdot \hat{H}_{3}^{*} \cdot \hat{\omega}_{r}' \cdot H_{4}^{*} \cdot \hat{\omega}_{r} \cdot \hat{H}_{5}^{*} \cdot \hat{\omega}_{a}')^{\mathrm{H}} \\
\cdot (\hat{H}_{5}^{*})^{\mathrm{H}} \cdot (\hat{\omega}_{r})^{\mathrm{H}} \cdot (\hat{H}_{3}^{*})^{\mathrm{H}} \cdot (\hat{\omega}_{r})^{\mathrm{H}} \cdot (\hat{H}_{2}^{*})^{\mathrm{H}} \cdot (\hat{\omega}_{a})^{\mathrm{H}} \cdot (\hat{H}_{1}^{*})^{\mathrm{H}} \cdot (\hat{\omega}_{r}')^{\mathrm{H}}, \\
= \begin{bmatrix} \sigma_{1} \cdot \varepsilon_{1} \cdot \zeta_{1} \cdot \zeta_{1} \cdot \zeta_{1} & 0 & \cdots & 0 \\ 0 & \sigma_{2} \cdot \varepsilon_{2} \cdot \zeta_{2} \cdot \zeta_{2} \cdot \zeta_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{N_{r}N_{a}} \cdot \varepsilon_{N_{r}N_{a}} \cdot \zeta_{N_{r}N_{a}} \cdot \zeta_{N_{r}N_{a}} \end{bmatrix} . \quad (39)$$

Therefore, the imaging operator **G** is an orthogonal matrix. Furthermore, $\hat{\Phi} \cdot \mathbf{G}$ obeys the RIP. Thus, $\Gamma^{-1}(\cdot) \cdot \Phi$ satisfies the RIP.

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