

A Simple and Ready to Use Code to Simulate Paraxial Beam Propagation

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Abstract: We provide a software package implementing the well-known beam propagation method to integrate the 1+1 and 1+2 dimensional nonlinear Schrödinger equation. It is useful to simulate optical beam propagation in the spatial and temporal domains. © 2021 The Author(s)

1. Introduction

Interactive software environments, also called virtual laboratories, can complement physical laboratories for student training [1]. Moreover, they are well adapted to the rapidly growing field of distance learning [2]. Clearly, they are also interesting for students that need to get used to the development of technical software for the computation and visualization of results. Taking this into account, we have developed a software library that provides a computational tool to students and young researchers dealing with laser beam propagation in non-trivial linear or nonlinear media. The present contribution is mostly based on our previous publication [3].

Specifically taking into account the possibility of using it in context of education and training of young scientists and engineers, the main goals that have guided the design of the software library are: wide accessibility, ease of installation and use, flexibility, simplicity and the possibility of having direct visualization of results. It is well adapted for the study of beam optics although it can also be of interest for courses in quantum mechanics, computational physics or nonlinear waves.

2. The numerical method and the software library

The propagation of laser beams is typically modelled by introducing the paraxial approximation in the wave equation that is derived from the Maxwell equations of electromagnetism, leading to what is sometimes called beam optics [4]. This results in an expression that is analogous to the Schrödinger equation typically used in quantum mechanics.

$$i \frac{\partial \psi}{\partial z} = -\frac{1}{2} \nabla^2 \psi + V \psi, \quad (1)$$

In the optical case, z is the spatial coordinate along the beam and V encodes the features of the refractive index profiles, which can include both linear and nonlinear terms. Solving this equation for different V and different initial conditions is important in optics and it is of interest for young students entering the field. Different numerical methods have been developed to address this problem, but we are interested in the beam propagation method [5,6], that relies on the computation of direct (\mathcal{F}) and inverse (\mathcal{F}^{-1}) Fourier transforms at each time step:

$$\psi(\mathbf{x}, t + dt) \approx \mathcal{F}^{-1} [e^{\frac{i}{2} dt (2\pi i \mathbf{k})^2} \mathcal{F} [e^{-i dt V} \psi(\mathbf{x}, t)]], \quad (2)$$

We have created a Python-based open source software library to integrate this equation in 1+1 or 1+2 dimensions [3]. The user can insert an initial condition, a potential and several inputs that define the numerical computation such as the number of points of the grid. The computation is performed and the graphical output is generated automatically. The library could be generalized for the 1+3 D case to study spatio-temporal beam dynamics, although that would require further efforts to provide adequate graphics (see for instance [7], [8]). The library has a modular structure and it can be convenient for undergraduate students interested in performing simple virtual experiments, for instance just changing some parameters and comparing the outcomes or as a starting point for more advanced students who want to explore different types of linear or nonlinear refractive indices and deal with different issues of the numerical algorithm. It should also be noticed that a simple modification of the code

to introduce propagation in imaginary time [9] can allow the user to compute ground-state propagation-invariant profiles for a given potential.

The library can be downloaded from <https://github.com/pyNLSE/bpm>, where extensive documentation is also provided, together with guidelines for getting started in Python for users that may not be acquainted with it. There are also twenty examples of 1D and 2D cases, both linear and nonlinear that can be used by students to understand the basic features of the code in order to be able to later address different problems.

3. Examples

In figure 1, we depict the outcome of two of the examples provided with the software library. The first example is an interesting example of scattering in 1+1 dimensions. For $V = -\frac{s(s+1)}{\cosh^2 x}$, the potential becomes reflection-less when s is an integer, as it can be appreciated in the figure. It is an illustrative exercise for a student to repeat the computation with different values of s . On the right, we consider a 1+2 dimensional case with a cubic-quintic nonlinearity such that there are solitons with liquid-like properties [10]. When the soliton droplet collides with a barrier, its behavior qualitatively resembles that of a drop of water falling on a solid surface. The graphical displays shown in the figure have been chosen among the different figures automatically generated by the Python code.

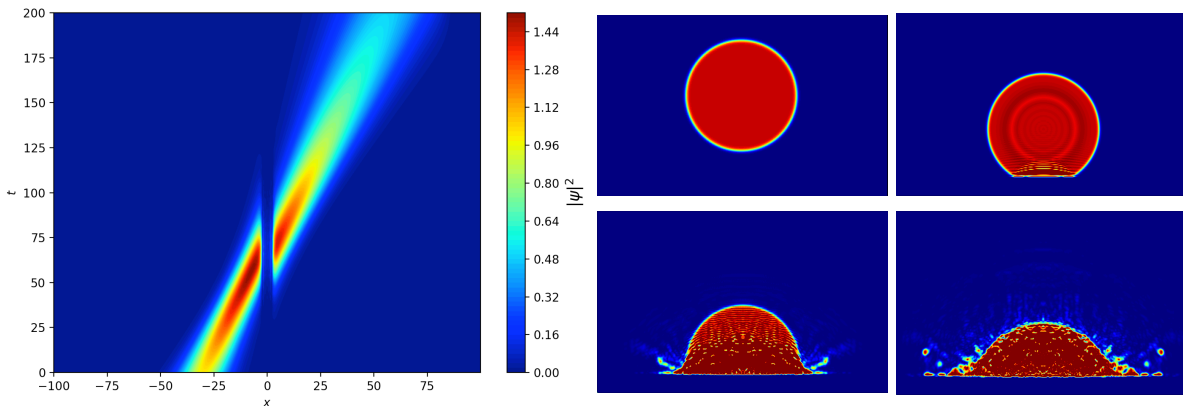


Fig. 1. On the left, the output of the (linear) scattering of a beam by a reflection-less potential. On the right, the breaking of a liquid-like soliton in a cubic-quintic medium due to its collision with a barrier.

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