Computer Generated Holograms (CGH) realization: the integration of dedicated software tool with digital slides printer

V. Guarnieri(+), F. Francini(*)

(+)CEO-Centro di Eccellenza Optronica
Largo E. Fermi, 6 I-50125 Florence, Italy
(*)Istituto Nazionale di Ottica
Largo E. Fermi, 6 I-50125 Florence, Italy

ABSTRACT

Last generation of digital printer is usually characterized by a spatial resolution enough high to allow the designer to realize a binary CGH directly on a transparent film avoiding photographic reduction techniques. These devices are able to produce slides or offset prints. Furthermore, services supplied by commercial printing company provide an inexpensive method to rapidly verify the validity of the design by means of a test-and-trial process. Notably, this low-cost approach appears to be suitable for a didactical environment.

On the basis of these considerations, a set of software tools able to design CGH's has been developed. The guidelines inspiring the work have been the following ones: a) ray-tracing approach, considering the object to be reproduced as source of spherical waves; b) Optimization and speed-up of the algorithms used, in order to produce a portable code, runnable on several hardware platforms.

In this paper calculation methods to obtain some fundamental geometric functions (points, lines, curves) are described. Furthermore, by the juxtaposition of these primitives functions it is possible to produce the holograms of more complex objects. Many examples of generated CGHs are presented.

Keywords: Computer Generated Hologram (CGH), diffractive optics, ray-tracing

1. INTRODUCTION

Thin layers of transparent films, whose optical path is controlled by means of thickness or refractive index variations, are used to make Diffractive Optical Elements (DOE). The way in which DOE works is simplified in Figure 1.

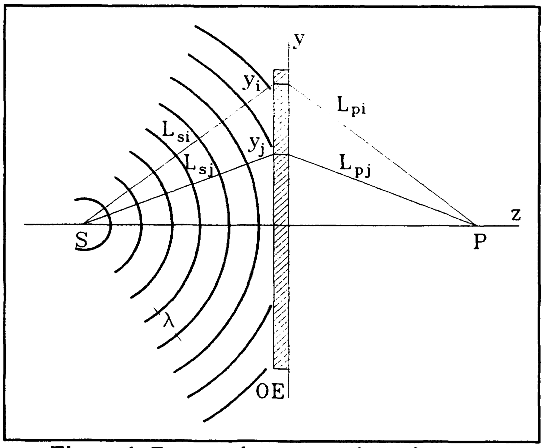

Figure 1. Ray-tracing generation of a CGH

In the figure, two beams start from the source S and attain the optical element OE in *Yi* and *Yj* . They supply contribution to the image P of S only if they have in P the same phase. The elements of OE located in *Yi* and *Yj* work as phase advancer elements. In particular, the following equation must be valid:

$$\varphi_S + \varphi_{OE} - \varphi_P = 0 \tag{1}$$

where φ_S, φ_{OE}, φ_P are the phase differences respectively in the optical path L_{sj}, in OE and in L_{pj}.
Thus, the phase difference in an arbitrary point of OE is given by:

$$\varphi_{OE_j} = \frac{2\pi}{\lambda} L_{pj} - \frac{2\pi}{\lambda} L_{sj} \tag{2}$$

Eq. (2) assume the initial phase of rays coming from S equal to zero.

A CGH (Computer Generated Hologram) is an hologram generated by a computer and fabricated under computer control. It is produced by recording the interference pattern formed by two coherent optical wavefronts, referred as reference wave and object wave. The computer calculates the interference pattern that would be produced by object and reference function. The information is sent to a device recording the data as either grey or phase levels in an optically transparent film.
The phase delay in each pixel of the CGH is given by Eq. 2 and it is used to obtain the intensity of interference fringes. The fringe intensity in a point *Yj* of the hologram plate, is the well known cosine law expressed, apart from some constants, by the simplified equation:

$$I(y_j) = 1 + \cos(\varphi_{OE_j}) \tag{3}$$

This equation concerning a single image point, may be extended for complex images representation. The calculation methods use Fourier-Fresnel transforms[1,2,3,4,5,6,7] or ray-tracing algorithms. In both cases the image to be reproduced is considered as a light source whose phase relations with the reference beam must be calculated in the hologram plane. If the object is far from the hologram plane or it is produced in the focal plane of a lens, a Fast Fourier Transform technique is used. When the object is in a plane near the hologram, Fresnel transform or ray-tracing algorithms will be used.
In this paper, only the ray-tracing approach is considered, because of its didactical evidence and simplicity. Furthermore, if the image is considered formed by several single points, for each of them a separate phase map is obtained. Detailed information about this procedures are discussed in following sections.

2. RAY-TRACING METHODS

In a set of points forming an extended object, the corresponding intensity pattern is given by the superposition of each pattern. The total intensity pattern I_{tot} comprises three components as shown in the following equation:

$$I_{tot} = |E_0|^2 + |E_R|^2 + 2R_e\{E_0 E_R^*\} \tag{4}$$

The total object field is represented by:

$$E_0 = \sum_{p=1}^{Npts} \frac{a_p}{r_p} \exp(i\phi_p) \qquad (5)$$

N_{pts} is the numbers of object points, each of them has an associated real-value magnitude a_p and phase $\phi_p = kr_p$ where r_p is the distance from the object to the hologram.

The reference beam in the case of a point source is:

$$E_R = \frac{a_R}{r_r} \exp(i\phi_R) \qquad (6)$$

The first and the second terms of Eq. (4) are related respectively to the object waves and to the reference wave intensity. Since the reference beam is stronger than object beam, the first term is negligible. The second term represents a bias increasing its intensity throughout the hologram.
The third term contains all the information needed to reconstruct the image, it is numerically simpler to compute and has the advantage of containing neither object self-interference nor bias components[8,9]. After some simplifications, the final equation is:

$$I \propto K_{off} + \sum_{p=1}^{Npts} \frac{a_P}{r_p} \cos[\phi_P - \phi_R] \qquad (7)$$

This is the basic expression used to calculate the hologram fringes. K_{off} is a normalization factor used to make positive all pixel values.

3. RAY-TRACING GENERATED ELEMENTARY FUNCTIONS

The following paragraphs show the analytic definition of some elementary functions whose expression is representable with ray-tracing methods [10,11, 12, 13, 14].

3.1 Image of a point

If the coordinate of the image plane and of the hologram plane are respectively (η, ξ) and (x, y), a collimated laser beam impinging on the hologram plane at an angle θ varies its phase value according to the following equation:

$$\phi_R = \frac{2\pi}{\lambda} x \cos\theta \qquad (8)$$

In the case of a plane wave normally incident on the hologram plane, ϕ_R, is a constant. The phase difference on each hologram pixel is determined by the expression:

$$\varphi_{xy} = \frac{2\pi}{\lambda} \sqrt{(y-\xi_0)^2 + (x-\eta_0)^2 + z_0^2} \qquad (9)$$

where ξ_0, η_0, z_0 are the image point position .

3.2 Image of a line with rotation α and tilt γ

$$\varphi_{xy} = \frac{2\pi}{\lambda}\sqrt{(y'-\xi'_0)^2 + (x'-\eta'_0)^2 + z'^2_0} \tag{10}$$

where:

$$\begin{aligned}
\xi_0' &= \xi_0 \cos\alpha - \eta_0 \sin\alpha \\
\eta_0' &= \xi_0 \sin\alpha + \eta_0 \cos\alpha \\
x' &= x\cos\alpha - y\sin\alpha \\
y' &= (x\sin\alpha + y\cos\alpha)\cdot\left(1 - \frac{L}{L_{hol}}\right) \\
z_0' &= z_0 - y'\tan\gamma
\end{aligned}$$

where ξ_0 , η_0 e z_0 are the coordinates of the middle point of the line. L is the line length and L_{hol} represent the hologram side length. Rotation and tilt of the line are respectively α and γ.

3.3 Image of an helicoid with axis parallel to y hologram axis

$$\varphi_{xy} = \frac{2\pi}{\lambda}\sqrt{(x - x_{hel})^2 + y^2 + (z - z_{hel})^2} \tag{11}$$

where:

$$\begin{aligned}
x_{hel} &= r_{hel}\cos(n\cdot\beta) \\
z_{hel} &= r_{hel}\cos(n\cdot\beta) \\
\beta &= 2\pi + \frac{\pi y}{L_{hol}}
\end{aligned}$$

n is an integer, it changes the helix step. r_{hel} represent the helix radius.

3.4 Aspherical surface

A wide variety of lenses can be holographically generated. The optical paths differences characterizing an aspherical surface are expressed by optical path variations z_{xy}[15, 16]; the corresponding phase differences are defined by $\varphi_{xy}=2\pi z_{xy}/\lambda$, where z_{xy} is given from the following expression:

$$z_{xy} = \frac{cS^2}{1+\left[1-(K+1)c^2S^2\right]^{1/2}} + A_1S^4 + A_2S^6 + A_3S^8 + A_4S^{10} \tag{12}$$

where:

$$S^2 = x^2 + y^2$$
$$c = 1/r.$$

r is the curvature radius, A_1, A_2, A_3, A_4 are the aspheric deformation constants of the surface; K is the eccentricity coefficient. Its value determines the conic surface type according to the following table:

$$\begin{cases} K < -1 & \text{Hyperboloid} \\ K = 1 & \text{Paraboloid} \\ -1 < K < 0 & \text{Ellipse rotated about its major axis} \\ k = 0 & \text{Sphere} \\ K > 0 & \text{Ellipse rotated about its minor axis} \end{cases}$$

3.5 Axicon

The cone-shaped axicon can be efficiently represented by approximating it with an hyperboloid of suitable curvature[17]. The equation representing K and c are given by:

$$K = -\left(1 + \tan^2 \theta\right) < -1 \qquad (13)$$
$$c = \frac{1}{(K+1)b}$$

θ e b are the cone aperture and the distance between the cone and hyperboloid vertexes.

3.7 Anamorphic aspherical surface

$$z_{xy} = \frac{c_x x^2 + c_y y^2}{1 + \left[1 - (1 + K_x)c_x^2 x^2 - \left(1 + K_y\right)c_y^2 y^2\right]^{1/2}} + \psi(x,y) \qquad (14)$$

where c_x and c_y are the curvatures along x ad y axis respectively and K_x and K_y represent the eccentricity along the coordinate axes. $\psi(x,y)$ is a function including the bidimensional 4th, 6th, 8th, 10th order deformation from conic [18].

5. THE SOFTWARE TOOLS

A severe limitation concerning the practical generation of CGH is inherently represented by the complexity of calculations required from their definition. The use of Eq. (7) strongly decreases the computing time without sensible loss of accuracy, but with the use of a last-generation personal computer is possible to obtain reasonable execution times, extending the application fields.
The dedicated software tools have been developed taking into consideration portability on many platforms, from PC's to workstations. With regard to PC's, two versions have been released, running under Linux and DOS respectively.

The Unix version makes is provided of an X11 interface, developed by Forms Library[19]. The library is compatible with ANSI-C compilers and it is available for many Unix platforms. We tested the procedure on HP-UX 9.0 and Linux 2.0.27 operating systems. A sample of the input screen is shown in Figure 2.

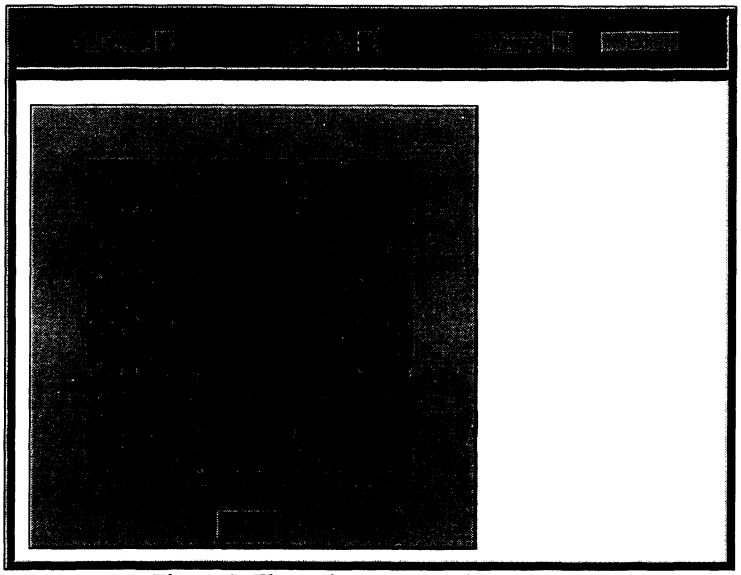

Figure 2. The main procedure input screen

In order to maintain as long as possible the advantages of the 32-bit architecture, source code has been ported on the DJGPP compiler, to produce a DOS executable. In this case, a more simple user interface has been developed, based on alphanumeric screens.

The procedure allows the user to combine the available primitives function described in Section 3 (points, lines, curves...) composing more complex objects, defined trough their components.

For each of them, the corresponding hologram is stored in a temporary buffer; the final hologram is obtained as normalized sum of the partial component holograms.

Each buffer is a double precision array containing the amplitude values of the hologram. The size of the holograms is limited by the amount of the available RAM. A typical working size is 1600x1600 pixel.

In order to generate a binary CGH, good results were obtained thresholding the resulting grey-levels pattern. In the described procedure this task is accomplished by means of a dedicated algorithm allowing the user to specify the threshold method.

The resulting raster file is exported to the more common graphic file formats (TIFF and TGA).

In most cases, the computing times are satisfactory from a practical point of view. Table 1 shows a comparison between different platforms in typical applications.

Computer	Hologram type			
	4 point hologram	2 lines hologram	aspheric lens	32 points helicoid
486 DX2 66 MHz	18	25	35	135
Unix workstation	8	12	15	60

Table 1. Computing times on different platforms in seconds for 800x800 pixel holograms.

6. PRINTING TECHNIQUES

The basic assumption underlying this work is the possibility to print the CGH's with a low cost device. Several printing devices have been investigated from this point of view. To realize binary holograms, an initial choice can be a commercial laser printer. These devices are commonly characterized by a spatial resolution of 600 dpi (dots per inch), corresponding to 40 μ about. This value is not good enough to get a good resolution CGH. Enhanced laser printers have a resolution of 1200 dpi, that is ~20 μ of resolution, with a slightly added purchasing cost.

Albeit it is possible to get some results with such printers, more resolution is requested to have a practically usable CGH. A good tradeoff between cost and performances is represented by the use of two kind of devices: digital slide printers and offset printing systems.

The last generation of digital slide printers allows the user to print on a photographic film with a resolution up to 4096 points. At 1 pixel/point, by choosing the right orientation, it is possible to achieve an equivalent resolution of 5.8 μ. Anyway, the obtained results by means of commercially available photographic films show a lower resolution due to emulsion grain.

A better alternative to produce low-cost CGH's is given by offset print. The typical resolution is 3600 dpi, corresponding to a pixel size of 7 μ about. With such devices it is possible to produce a general purpose hologram, i.e. a CGH able to diffract the incident light at angle not too close to the zero-order beam (tipically 3 degrees).

The offset technique is usually employed for high-quality typographic prints recording binary CGH's on acetate sheet. The typical hologram size obtained with this resolution is 11.2 mm x 11.2 mm corresponding to an image of 1600x1600 pixels. These techniques produce amplitude holograms whose efficiency is around 6%. One can use a standard optical hologram copying setup to copy the amplitude hologram into a phase medium, such as photopolimer (by DuPont) or photoresist, increasing the total efficiency.

7. RESULTS

Some holograms of the described objects before the binarization procedure are shown in the next figures. In order to have a good-quality reproduction, their size has been reduced to 400x400 pixels, working wavelength being 632 nm.

Figure 3 represents an anamorphic aspheric lens generated according to Eq. (14). Figure 4 shows the CGH of a 2 mm length line rotated of 45° and tilted of 0.1°. The image is formed 400 mm far from the optical element. Figure 5 shows the CGH of two segments normally incident in their centre. Figure 6 shows the CGH of four spots generated by a multifocal lens. Figures 7a and 7b show a 16-points cylindrical helix along with the corresponding CGH. Axes η, ξ, z are relative to the points position in the image space.

8. CONCLUSIONS

An integrated software tool able to generate binary CGH's with ray-tracing methods was developed. The procedure allows the user to design and test CGH's controlling their geometrical and physical parameters. The source code was successfully ported from workstations to PC's and compiled under DOS, Linux and Unix platforms. The computing times are reasonable when considering few centimeters holograms. The output format is compatible with most of the printing devices.

With regard to the physical realization of the holograms, digital slides printer and offset technique were tested. Better results were obtained with offset technique. Although the obtainable resolution allow the reconstruction of the objects mostly in proximity of the optical axis, their visibility is very good. The

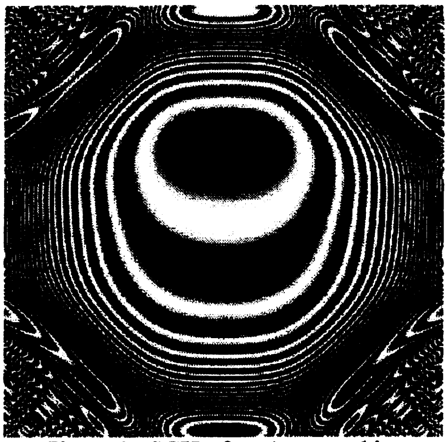

Figure 3. CGH of an Anamorphic Aspheric lens

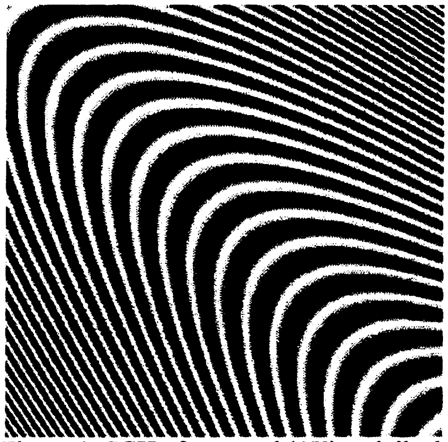

Figure 4. CGH of a rotated (45°) and tilted (0.1°) line

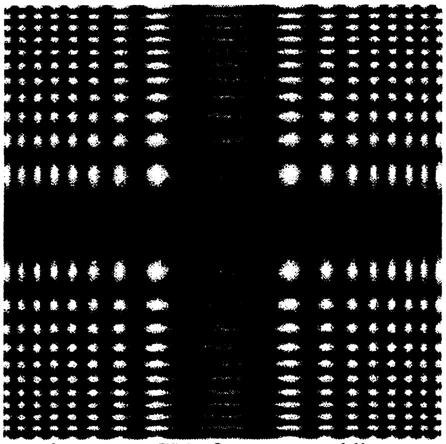

Figure 5. CGH of two crossed lines

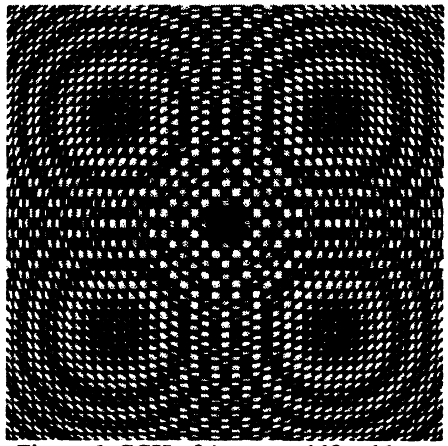

Figure 6. CGH of 4-spot multifocal lens

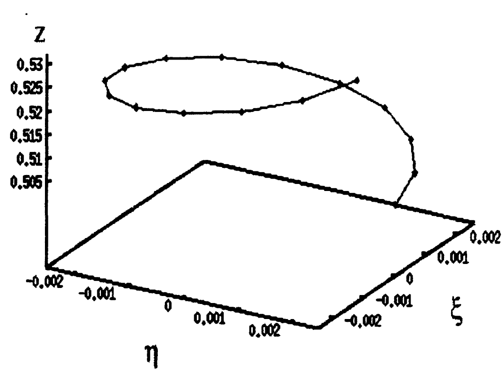

Figure 7a). A cylindrical helix made of 16 points

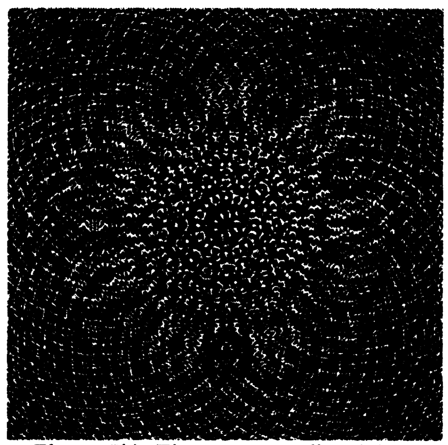

Figure 7b). The corresponding CGH of the cylindrical helix

presented technique represent the best practical trade-off from cost and performances point of view. Therefore, we suggest the use of this techniques mainly in training and educational environments.

9. REFERENCES

1. B. R. Brown, A. W. Lohmann, "Complex Spatial Filtering with Binary Mask," *Appl. Opt.* **5**(), 967-969 (1967).
2. W-H. Lee, "Binary Synthetic holograms," *Appl. Opt.* **13**(7), 1677-1682 (1974).
3. W-H. Lee, "Computer generated holograms: techniques and applications," in *Progress in Optics,* E. Wolf Ed., Vol. 16, 119-232, SPIE, North Holland, Amsterdam (1978).
4. A. Kozma, D. L. Kelly, "Spatial Filtering for Detection of Signals Submerged in Noise," *Appl. Opt.* **4**(4), 387-392 (1965).
5. G. Tricoles, "Computer generated holograms: an historical review," *Appl. Opt.* **26**(20), 4351-4360 (1987).
6. W-H. Lee, "Binary computer-generated holograms," *Appl.Opt.* **18**(21), 3661-3669 (1979).
7. S. Morucci, P. Noirard and J. C. Grossetie, "Improvement of Fresnel computer-generated holograms," *Opt. Comm.* **134**(1), 25-30 (1997).
8. A. D. Stein, Z. Wang and J. S. Leigh Jr., "Computer-generated holograms: A simplified ray-tracing approach," *Computer in Physics* **6**(4), 389-392 (1992).
9. M. Lucente, "Interactive Computation of Holograms using a Look-up Table," *J. of Electronic Imaging* **2**(1), 28-34 (1993).
10. X. Chen, J. Huang and E. Loh, "Computer Generated holograms Using Personal Computers," *Optics and Laser in Engineering* **7**(1), 83-90 (1987).
11. R. Silvennoinen, J. Räsänen and T. Jääskelainen "A novel optical interconnection method," *Opt. Comm.* **93**(5), 289-293 (1992).
12. C. Frère, D. Leseberg and O. Bryngdahl, "Computer-generated hologram of three-dimensional objects composed of line segments," *J. Opt. Soc. Am. A* **3**(5), 726-730 (1986).
13. D. Leseberg, C. Frère, "Computer generated hologram of 3D-object composed of tilted planar segments," *Appl. Opt.* **27**(14), 3020-3024 (1988).
14. D. Leseberg, "Computer generated holograms: cylindrical, conical, and helical waves," *Appl. Opt.* **26**(20), 4385-4390 (1987).
15. D. Malacara ed., *Optical Shop Test*, Wiley, New York, 1992
16. G. Schulz, "Aspherical surfaces", in *Progress in Optics*, E.Wolf Ed., Vol. 25, 349-416, SPIE, North Holland, Amsterdam (1978).
17. J. H. McLeod, "The Axicon: A New Type of Optical Element,", *J. of Optical Society of America* **44**(8), 592-597 (1954).
18. *Code-V Reference Manual*, CODE V Version 8.10, Optical Research Associates, Pasadena (1996).
19. T. C. Zhao, M. Overmars, *Forms Library-A Graphical User Interface Toolkit for X*, 1995.