# Analytical method for calculating the leakage between river and groundwater

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## ABSTRACT

With the aim of exploring the dynamic process of the relationship between river water and groundwater, a mathematical model for the unstable seepage process of groundwater under sudden changes in river water level is established. Taking the No. 5 rubber dam reservoir area of the Bahe River as an example, the variation of groundwater level around the reservoir area after the rubber dam storage is analyzed, and the relationship between the amount of groundwater replenished by the river and time is simulated, Calculate the changes in groundwater level around the Bahe River and the amount of seepage in the Bahe River after the rubber dam is filled with water.

**Keywords:** Bahe River, analytical method, river water, groundwater, leakage amount

## 1. INTRODUCTION

The main methods for studying the interaction between river water and groundwater include outdoor and indoor experimental methods, dynamic data analysis methods, basic flow cutting methods, hydrochemical methods, and groundwater dynamics methods[1-6]. Heij[7] used hydrological dynamic data to calculate the replenishment amount between rivers and groundwater and found a linear relationship between river water level and the infiltration rate of river water. The basic flow cutting method cuts the basic flow based on the flow process line and is often used to estimate the conversion amount between river water and groundwater[8]. The hydrochemical method analyzes the water quality of rivers and groundwater, qualitatively identifies the hydraulic relationship between the two, analyzes the sources of groundwater recharge, and determines the proportion of water from various sources. The groundwater dynamics method is the basis for quantitatively determining the exchange rate between rivers and groundwater and can be divided into analytical methods[9] and numerical simulation methods[10].

This article establishes an analytical solution for the unstable seepage process of groundwater under sudden changes in river water level and takes the No. 5 rubber dam reservoir area of the Bahe River as an example to calculate the changes in groundwater level around the Bahe River and the leakage amount of the Bahe River after the rubber dam is filled with water.

## 2. MATHEMATICAL MODEL

In a homogeneous, isotropic, horizontally impermeable aquifer with a permeability coefficient of $K$ and a thickness of $h$, the one-dimensional differential equation for groundwater movement perpendicular to the unit width of the river channel is

$$\frac{\partial\left(Kh\frac{\partial h}{\partial x}\right)}{\partial x}=\mu\frac{\partial h}{\partial t} \tag{1}$$

When the variation of groundwater level is much smaller than the thickness of the aquifer, $Kh$ in equation (1) can be approximately replaced by $Kh_m$, where $h_m$ is the average groundwater level at the beginning and end of the time period. At this point, equation (1) can be simplified as

$$\frac{Kh_m}{\mu}\frac{\partial^2 h}{\partial x^2}=\frac{\partial h}{\partial t} \tag{2}$$

Or,

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$$a\frac{\partial^2 h}{\partial x^2}=\frac{\partial h}{\partial t} \tag{3}$$

In the equation, a is defined as the pressure conductivity coefficient of the aquifer (groundwater diffusion coefficient).

Set the groundwater level (i.e. initial water level) $h_0$ as horizontal before the change in river water level. Taking the water level change value $s=h-h_0$ as the variable, establish a coordinate system as shown in Figure 1. The value of $s$ starts from the initial water level, rising positive and falling negative. When the water level of the river rises rapidly $\Delta H$, the basic equation and initial and boundary conditions for groundwater movement are

$$\begin{cases} a\frac{\partial^2 s}{\partial x^2}=\frac{\partial s}{\partial t} \quad 0<x<\infty, t>0 \\ s(x,0)=0 \quad 0<x<\infty \\ s(\infty,t)=0 \quad t>0 \\ s(0,t)=\Delta H \quad t>0 \end{cases} \tag{4}$$

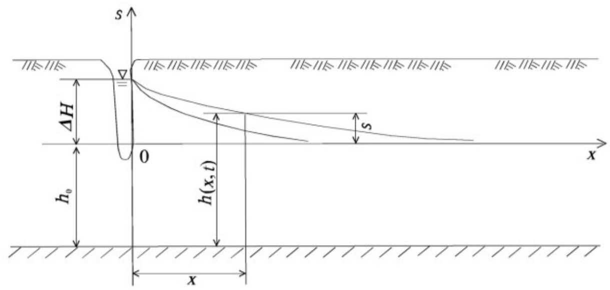


Figure 1. The movement of groundwater under the influence of rivers.

By using the Laplace transform to solve the above equation, the expression for the groundwater level rise value $s$ at point $x$ from the riverbed at time $t$ after the sudden rise of the river water level can be obtained as follows

$$s=\Delta H erfc(z)=\Delta H\overset{\frown}{S} \tag{5}$$

$$z=\frac{x}{2\sqrt{at}} \tag{6}$$

$$\overset{\frown}{S}=erfc(z)=\frac{2}{\sqrt{\pi}}\int_z^{\infty}\mathrm{e}^{-u^2}du \tag{7}$$

$$erfc(z)=1-erf(z) \tag{8}$$

$$erf(z)=\frac{2}{\sqrt{\pi}}\int_0^{z}\mathrm{e}^{-u^2}du \tag{9}$$

In the equation, $erf(z)$ is the error function; $erfc(z)$ is the residual error function.

If the pressure conductivity coefficient a of the aquifer is known, it is desired to calculate the sudden change in river water level at any distance $x$ and at any time $t$. The change in groundwater level caused by $\Delta H$ can be obtained by first calculating $z$ and then consulting the residual error function table $\overset{\frown}{S}(z)$. The value of $s$ can be determined by equation (5).

According to Darcy's law, when the water level suddenly rises, the infiltration rate $v$ of nearby groundwater is

$$v=-K\frac{\partial s}{\partial x}=\frac{\Delta HK}{\sqrt{\pi at}}\bullet e^{-\frac{x^2}{4at}}=K\frac{\Delta H}{x}\overset{\frown}{P} \tag{10}$$

The flow rate $q_x$ at any cross-section $x$ is

$$q_x = vh_m = \frac{\Delta HKh_m}{\sqrt{\pi at}} \bullet e^{-\frac{x^2}{4at}} = \frac{\Delta H\mu\sqrt{a}}{\sqrt{\pi t}} e^{-z^2} \tag{11}$$

When $x$=0, the single width flow rate $q_0$ of the river supplying groundwater to one side can be obtained as

$$q_0 = \frac{\Delta H\mu\sqrt{a}}{\sqrt{\pi t}} \tag{12}$$

The total amount of groundwater per unit width $U_x$ passing through any section x during time period $t$ is

$$U_x = \int_0^t q_x dt = 2\sqrt{at}\Delta Hierfc(\frac{x}{2\sqrt{at}}) \tag{13}$$

Taking $x$=0, we can obtain the total amount of groundwater supplied by the river $U_0$ to one side during time $t$.

$$U_0 = \int_0^t q_0 dt = 2\Delta H\mu\sqrt{at}ierfc(0) = 1.128\Delta H\mu\sqrt{at} \tag{14}$$

## 3. MODEL APPLICATION

Taking the Bahe No. 5 rubber dam reservoir area as an example, based on drilling data and pumping test data in the reservoir area, the thickness of the groundwater aquifer is $h_m$=43.8 m, the permeability coefficient is $K$=33.1 m/d, and the water yield is $\mu$=0.17, the pressure conductivity coefficient of the aquifer is $a$=8528.12 $m^2$/d. Assuming that the water level of the Bahe River rapidly rises by 3 meters in a short period of time after the rubber dam is filled with water, use equations (5) and (14) to calculate the changes in the groundwater level around the Bahe River and the amount of seepage in the Bahe River after the rubber dam is filled with water.

Figure 2 shows the time-dependent variation curve of groundwater level drop observed at different distances from the river. From the graph, it can be seen that at the same time, as the distance between the observation well and the river increases, the groundwater depth decreases. After 10 days of water storage in the rubber dam, the observed groundwater depth $s$ at distances of $x$=10, 30, and 50 m from the river were 2.94 m, 2.83 m, and 2.71 m, respectively. This indicates that the further away from the river, the smaller the impact of water storage in the rubber dam on its groundwater level. Under the same observation logging, as time goes on, the groundwater depth gradually increases, and the longer the time, the smaller the increase in depth. At $t$=1 d, 10 d, and 30 d, the groundwater drawdown of the observation well $x$=10 away from the river was 2.82 m, 2.94 m, and 2.97 m, respectively. This indicates that with the increase of time, the impact of rubber dam storage on its groundwater level gradually weakens for the same observation well.

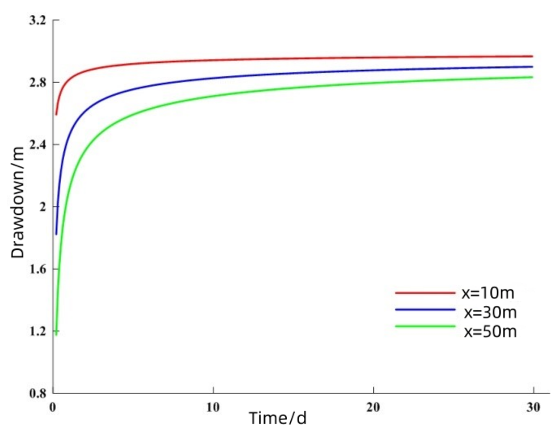


Figure 2. Time variation curve of groundwater level drawdown observed at different distances from the river.

Figure 3 shows the time-dependent curve of the flow rate of groundwater supplied by the river to one side. From the graph, it can be seen that as time goes on, the flow rate of groundwater replenished by the river gradually decreases, and the longer the time, the smaller the change in flow rate. At $t$=1 d, 10 d, and 30 d, the single width flow rate $q_0$ of the river supplying groundwater to one side is 26.57 $m^3/(m·d)$, 8.40 $m^3/(m·d)$, and 4.85 $m^3/(m·d)$, respectively. Using equation (14), it is calculated that within 30 days after the rubber dam is filled with water, the total single width $U_0$ of the river supplying groundwater to one side is 290.98 $m^3/m$. The length of the reservoir area of Bahe No.5 Dam is 2.8 km, and the leakage amount in the reservoir area after 30 days is 814750 $m^3$.

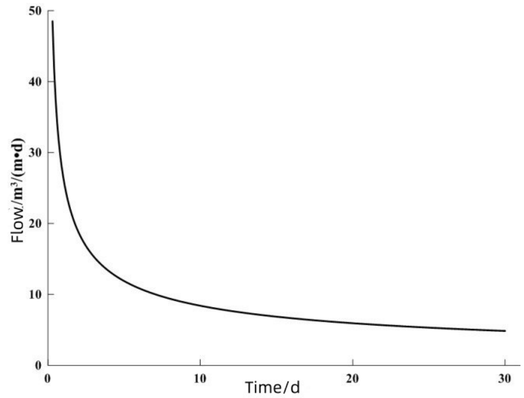


Figure 3. Time variation curve of groundwater flow rate replenished by rivers.

# 4. CONCLUSION

This article derives the analytical equation for unstable groundwater flow under sudden changes in river water level, and analyzes the time-dependent curve of groundwater level drop observed at different distances from the river. Without considering the weak permeable layer at the bottom of the riverbed, after the rubber dam is erected for 3 meters, the total single width $U_0$ of the water in the reservoir area of the No. 5 rubber dam of the Bahe River supplying groundwater to one side is 290.98 $m^3/m$.

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