Advanced Solitonic Metamaterial Structures under External Magnetophotonic Control

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ABSTRACT

Metamaterial research is an extremely important global activity that promises to change our lives in many different ways, including making objects invisible and having a very dramatic impact upon the energy and medical sectors of society. Behind all of the applications, however, lies the design of metamaterials and this can be led by elegant routes that include nonlinearity, waveguide complexity and structured light. The associated optical device formats often involve coupling to soliton behavior. Vortex formation is going to be a critical feature for future applications focusing attention upon the role of angular momentum in special metamaterial-driven light beams. In this context nonlinear diffraction must be assessed and some discussion of a magnetooptical environment will be included. Solitonic behavior of light beams will be mentioned, including what have now become known as Peregrine solitons.

Keywords: Metamaterial, nonlinearity, soliton, vortices, magnetooptics

1. INTRODUCTION

Metamaterials[1] lead to a completely new range of devices and, globally, the field has developed very well since the year 2000. The possibility of controlling metamaterial devices through magnetooptic environments is really exciting. Magnetooptics involves readily available materials in nanostructured form that can be easily embedded into plasmonic metamaterials[2]. Applications are now entirely accessible, therefore. In addition, it is becoming clear that isotropic metamaterials, with loss-free frequency windows[3-6], are within reach through composites containing certain types of nanospheres[7], especially since it is possible to make such composites using magnetooptic spheres. It is evident that new portals are emerging, implying new frontiers that need to be addressed in a lot more detail. Nonlinear systems are a vital part of this development, especially when coupled to tunability. Indeed, nonlinearity, leads to very special properties when dealing with negative index metamaterials, and may well be the way forward when addressing potentially devastating loss windows. The work reported here ranges over a range of possibilities, and it is emphasized that vortices and vortex solitons[8,9] can be important, especially if they are coupled to magnetooptic control in plasmonic metamaterials.

2. SOLITONS AND VORTICES

In general, it can be arranged, experimentally, for nonlinearity to be so strongly power-driven that solutions to Maxwell's equations cannot assume that the system can be described in terms of slowly-varying amplitude, a linear modal field and a fast phase variation. Fortunately, there is also another wonderful possibility, and that is to generate a family of *solitons*, for which separation into slowly-

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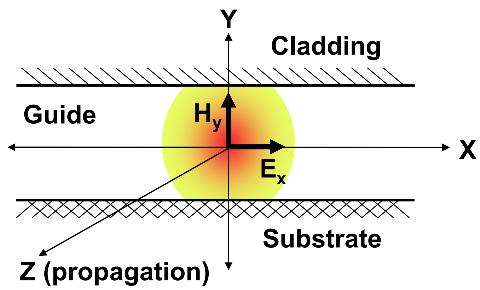
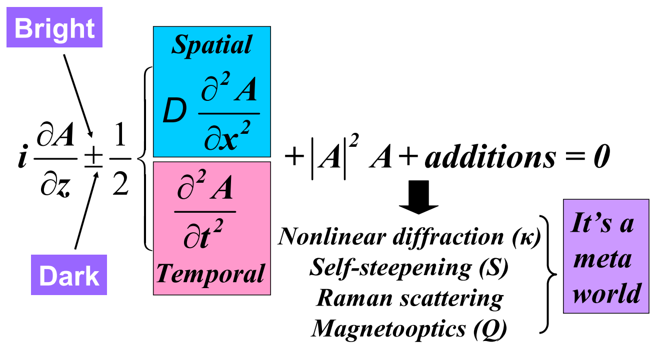


1(a)

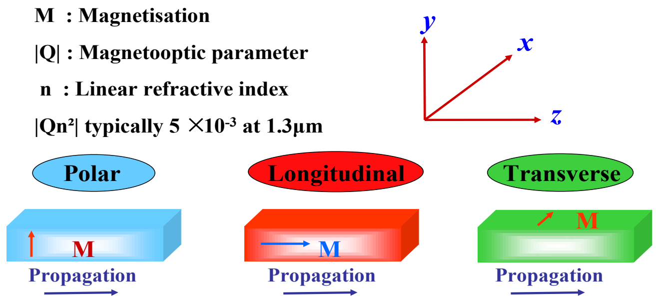


1(b)

Fig.1(a) Nonlinear planar waveguide[central core is nonlinear; cladding and substrate are linear] supporting stable one-dimensional spatial solitons, experiencing diffraction along the x-axis and associated with an electric field component E_x amd magnetic field component H_y.

Fig.1(b) This shows how the nonlinear Schrodinger equation, for complex amplitude A is built-up to model, bright or dark spatial solitons, or temporal pulse solitons. Note that D represents diffraction- management, or control. The influence through which the use of a metamaterial (It's a meta world)will have a strong influence are listed under additional effects.

varying amplitudes is, indeed, a fundamental step in the solution of Maxwell's equations, including control by nonlinearity. The soliton family includes spatial and temporal [10,11] varieties, and is governed by the what is called the nonlinear Schrodinger equation. Fig.1(a) shows an example of how one-dimensional spatial solitons can be generated in a planar guide. The basic one-dimensional nonlinear Schrodinger equation, for an amplitude A, is shown in Fig.1(b). The latter illustrates both the spatial and temporal cases and also shows that a parametric control of diffraction can be introduced through a parameter D. The latter can be effected by making the spatial solitons encounter an alternating layered [10] structure along the z-axis, the propagation direction. Fig.2(b) also indicates that additional terms can be added to the classic nonlinear Schrodinger equation to account for a range of controlling influences that include magnetooptics.An important, point to make, however, is that the additional influences are open to metamaterial impact. The latter includes extreme selections of permittivity and permeability.



$$\varepsilon = \begin{pmatrix} n^2 & 0 & -iQn^2 \\ 0 & n^2 & 0 \\ iQn^2 & 0 & n^2 \end{pmatrix} \quad \varepsilon = \begin{pmatrix} n^2 & -iQn^2 & 0 \\ iQn^2 & n^2 & 0 \\ 0 & 0 & n^2 \end{pmatrix} \quad \varepsilon = \begin{pmatrix} n^2 & 0 & 0 \\ 0 & n^2 & -iQn^2 \\ 0 & iQn^2 & n^2 \end{pmatrix}$$

Fig.2 Typical magnetooptic configurations

Fig.2 shows the classic magnetooptic configurations that are globally used. With respect to Fig.1 the magnetooptic material will be used as substrate, or as a cladding. Magnetooptic materials are easily created but, in the absence of an applied, field to create the magnetization **M** ,they consist of magnetized sub-volumes(domains) randomly oriented with respect to each other. The net result is little or no magnetization, so that an applied magnetic field is always needed to achieve an overall magnetization. Also of all material available it is films of the dielectric insulator yttrium iron garnet(YIG) that have attracted a great deal of attention over the years. Typically, the magnetooptic parameter is $O(10^{-4})$ so that, in the bulk, because the propagation wavenumber is $O(Q^2)$, the Voigt configuration displays little, or no, magnetooptic effect. This means that in the bulk, for the vortex case shown below, it is the Faraday configuration, which is selected because the wavenumber is O(Q). For the waveguide shown in Fig.1, it is the Voigt effect that is commonly used, however, with the guide configured in asymmetric form, using magnetooptic material as either in the substrate, or the cladding. Hence, for a structure like Fig.1, it will be TM polarized spatial solitons that are generated. The bottom line, for all the magnetooptic arrangements, is that it is a term of O(Q) that is needed as the additional term in Fig.1(b) in order to display magnetooptic effects.

$$A(z,t) = \left[\frac{(1-4a)\cosh(bz) + \sqrt{2a}\cos(\Omega t) + ib\sinh(bz)}{\sqrt{2a}\cos(\Omega t) - \cosh(bz)} \right] e^{iz}$$

Fig.3 Dimensionless breather solution of the standard nonlinear Schrödinger equation. z and t are normalized propagation distance and time. **Ω** : dimensionless modulation frequency. $0<a<1/2$: determines frequencies with gain. b: determines instability growth.

In 1983 Peregrine predicted the existence of a soliton that is both localized in time and in space. It is a solution of the nonlinear Schrodinger equation that formats what are called oceanic rogue waves. Fortunately, deep ocean waves have a direct optical analog, so a study of optical pulses, for example, should lead to a direct observation of Peregrine solitons. In fact, they were brilliantly observed [12] for the first time in 2010 using an optical fibre system. It is interesting, therefore, to look at the impact of a metamaterial environment on such solitons and this will be briefly simulated below. Fig.3 shows a solution of the dimensionless nonlinear Schrodinger equation, for propagation down the z-axis and t is dimensionless time.

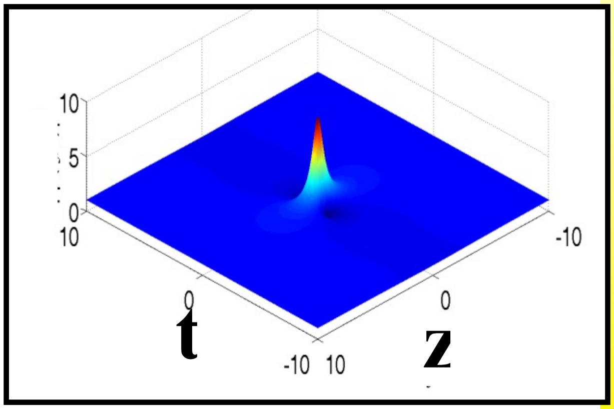


Fig.4(a) Peregrine soliton : $|A|^2$ as localized function of (t,z) in the limit a→1/2

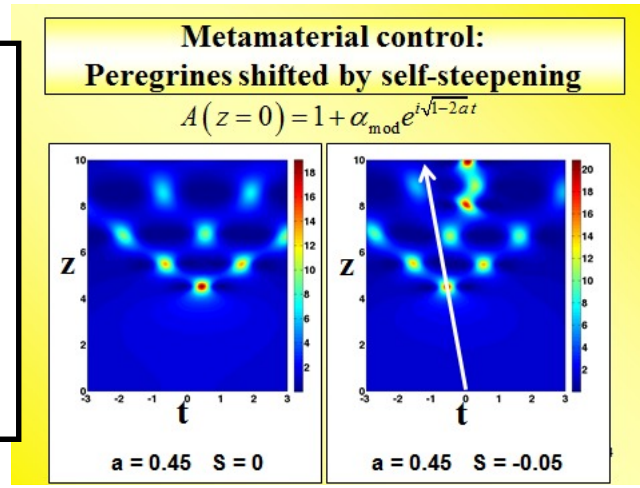


Fig.4(b) Peregrines near to the limit a→1/2 and metamaterial influence through self-steepening parameter S.

Fig.4 shows plots of plots of the Peregrine solutions to the nonlinear Schrodinger equation. In Fig.4(a) the plot is very close to the Peregrine single peak [soliton] solution as a→1/2, in order to avoid any computational singularities.. If the additional term, implied by Fig.1(b), is sought, so that metamaterial influence can be added to the Peregrine breather solutions then it will be the self-steepening that must be selected. This is quantified with the parameter S, in Fig.4(b). In order to make the role of S much more visually apparent, a value of a close, but not equal, to 0.5 is selected. So Fig.4(b) uses a= 0.45. Now it is not just one spike that appears, as in Fig.4(a), but several and it is apparent that the metamaterial influence can be exerted through S. In this, typical, example S=-0.05. It is concluded that the Peregrines can be moved quite readily over the (z,t) plane, which makes the use of a metamaterial to support the Peregrines a potentially useful device feature.

Moving on now to discuss vortices in metamaterials, it is possible to arrange for the basic equations contain the magnetooptic parameter, Q introduced in Fig.2 and because diffraction will take place along both the x and y direction that are perpendicular to the propagation axis, it will be the Faraday configuration, therefore, that will be used. Physically, this involves a constant applied magnetic field, directed along the propagation direction (z-axis). The Faraday effect actually couples the vortex amplitudes in the x and y directions and, after some interesting algebra, this kind of magnetooptic environment leads to the following coupled equations for the complex, normalized, amplitudes A_x and A_y of a vortex.

$$\begin{aligned} i\frac{\partial A_x}{\partial Z}+\frac{D}{2}\nabla_\perp^2 A_x-i\upsilon(x)\psi_y+\left(\left(|A_x|^2+|A_y|^2\right)A_x+f\left(A_x^*A_y-A_xA_y^*\right)A_y\right)+\kappa\frac{\partial}{\partial x}\left(\frac{\partial P_{0x}}{\partial x}+\frac{\partial P_{0y}}{\partial y}\right)=0 \\ i\frac{\partial A_y}{\partial Z}+\frac{D}{2}\nabla_\perp^2 A_y-i\upsilon(x)\psi_x+\left(\left(|A_x|^2+|A_y|^2\right)A_y+f\left(A_x^*A_y-A_xA_y^*\right)A_x\right)+\kappa\frac{\partial}{\partial y}\left(\frac{\partial P_{0x}}{\partial x}+\frac{\partial P_{0y}}{\partial y}\right)=0 \end{aligned} \qquad (1.1)$$

$$P_{0x,y} = \pm\left(\left|A_x\right|^2 + \left|A_y\right|^2\right)A_{x,y} \pm f\left(A_x^* A_y - A_x A_y^*\right)A_{y,x}\,, \tag{1.2}$$

where $A_{x,y}$ are the normalized electric field components along x and y and the latter are normalized by measuring in w units, where w is the effective width of the vortex. The Z-coordinates are also the normalized version obtained by using the units kw^2, where k is the effective wavenumber. The normalized magnetooptic function $\upsilon(x)$ accounts for the role of the applied magnetic field. Using the notation shown in Fig.2, $\upsilon(x) = \frac{1}{2}Q(x)\frac{\omega^2}{c^2}n^2w^2$, in which the magnetooptic parameter Q is made into a function of the coordinate x. An important point, is that if Q is simply a constant it will do nothing but add an additional phase shift to the vortex soliton. It is necessary to adopt a spatial form that causes the magnetization to decline way in both directions from x=0. κ is the nonlinear diffraction coefficient. The latter is a very interesting outcome for the nonlinear Schrodinger equation and is normally not discussed because it ranks as a fifth-order correction to the usual third-order nonlinearity. Nevertheless, it can play a role in terms of competing strongly to dominate over any nonparaxiality that may occur and it may also be competitive when using narrow beams. *D* is still a measure of linear diffraction-management. Note that since $\kappa = 1\big/\left(\varepsilon(\omega)\mu(\omega)w^2\right)$,where *w* is the effective diameter of the vortex, $\varepsilon(\omega),\mu(\omega)$ bring in, very strongly, the full properties of the metamaterial. Either self-focusing, or self-defocusing, can be selected at this stage, even though it is known that it is self-defocussing that leads to azimuthally stable vortex solitons. Also, a general third-order nonlinear polarization is assumed for which f=0, 1/3, or 1, depending upon whether thermal, electronic or molecular nonlinearity is going to be present.

The management of the linear diffraction can be achieved by placing, along the z-axis, alternating layers of positive phase and negative phase materials, each of them being matched to each other with a graded-index that eliminates any reflections. Fig.5 shows a, relatively, simple simulation that reduces (1.1) to a single equation by eliminating any magnetooptic influence, through setting the externally applied magnetic field to zero, and recognizing that the nonlinear diffraction has only a small role to play. It shows that the natural rotation of a vortex-soliton pair can be manipulated through the choice of diffraction-management parameter, D. D=100% is the normal diffraction environment. Anything less than this, lends itself to the possibility of controlling the vortex rotation, as can be seen quite clearly for the arbitrary choice of proapagation distance $Z=Z_1$.

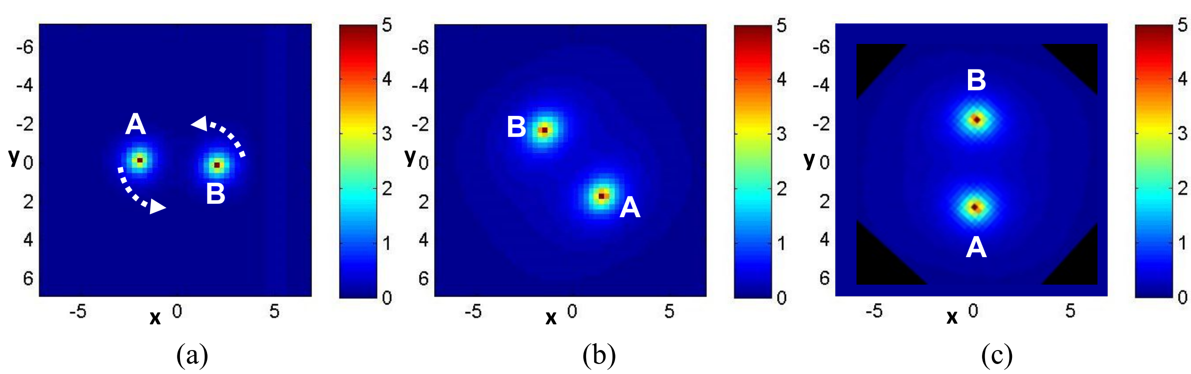


Fig. 5. (a) A vortex pair input. (b) A vortex pair in a D = 100% medium, after a propagation distance, Z_1=10. (c) A vortex pair in a D = 50% medium, after a propagation distance, Z_1=10.

The application of a magnetic field, in order to create a magnetooptic environment, changes the potential well, in which the vortex resides, very significantly. This action can make the vortex unstable. If this is engineered then the vortex may split up into two bright spatial solitons. This is demonstrated in Fig.6. The metamaterial influence is present through the choice of the linear diffraction-management parameter D=20%. The magnetooptic action is simply to switch on the applied magnetic field, given that the correct magnetooptical materials have been deployed. In Fig.6 the sign of the magnetooptic function ,defined earlier as, $\upsilon(x)$, is is selected at its maximum value, D=20% and an arbitrary propagation distance Z=60 is used.

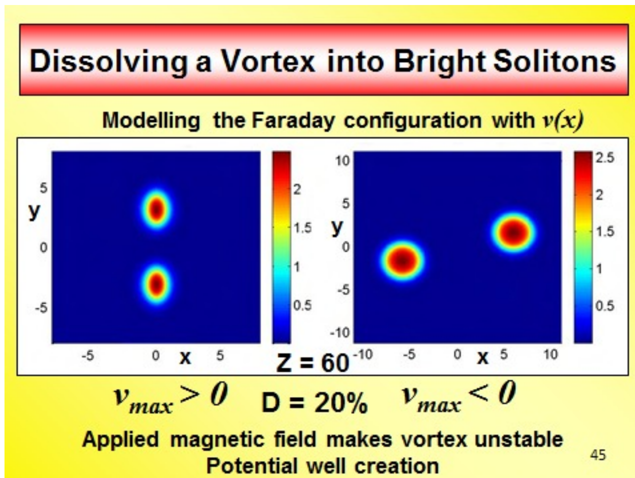


Fig.6 Demonstration of how magnetooptical influence can cause a vortex to split into two bright solitons.

3. CONCLUSIONS

It is emphasized that metamaterials will lead to a completely new range of devices. A nonlinear planar waveguide is discussed and it is shown that additions to the standard nonlinear Schrödinger equation will contain features such as nonlinear diffraction, self-steepening, Raman scattering and magneto optics. A a discussion of solitonic guides is given and it is shown that Peregrine solitons can be elegantly controlled with self-steepening. A detailed discussion of diffraction-managed optical vortices is given and it is demonstrated that important design metamaterial influence can be exercised over vortex propagation, especially vortex pairs. The fact that the applied magnetic field used in exercising magnetooptic influence over vortices changes the effective potential well is shown to force a vortex to dissolve into bright spatial solitons. Fundamentally, the paper is designed to initiate discussions of how metamaterial devices can be controlled through magnetooptics and through the use of highly structured light, such as vortices.

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