Diffraction Operators in Paraxial Approach

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ABSTRACT

Nowadays, research in the field of science education points to the creation of alternative ways of teaching contents encouraging the development of more elaborate reasoning, where a high degree of abstraction and generalization of scientific knowledge prevails. On that subject, this research shows a didactic alternative proposal for the construction of Fresnel and Fraunhoffer diffraction concepts applying the Fourier transform technique in the study of electromagnetic waves propagation in free space. Curvature transparency and Fourier sphere operators in paraxial approximation are used in order to make the usual laborious mathematical approach easier. The main result shows that the composition of optic metaxial operators results in the discovery of a simpler way out of the standard electromagnetic wave propagation in free space between a transmitter and a receptor separated from a given distance. This allows to state that the didactic proposal shown encourages the construction of Fresnel and Fraunhoffer diffraction concepts in a more effective and easier way than the traditional teaching.

Keywords: Optical Education, diffraction, Optical transformations, wave propagation, Huygens principle.

1. INTRODUCTION

Current research in science education, show the need to engage in educational practice a number of methods and techniques to ensure the development of critical thinking students predominantly from an increase in the level of reasoning used to understand content , and which are evident forms of scientific knowledge construction involving a higher degree of abstraction and generalization of it. This variation on the traditional way of teaching, the teacher requires finding quality learning and new creative strategies for their achievement in the terms initially exposed. This is to avoid the use of abstractions unmotivated predominantly associated with rote learning and liabilities acquired and decontextualized late in many cases, given the lack of significance for the learner [1]. It should be noted that the attitudes assumed by teachers to enter some content in the classroom in a traditional way, are characterized by the use of epistemological schemes that prevent the development of both creative attitudes in students, and in themselves when they make their educational work [2]. This of course in turn affects the lack of possibilities to form a more complex thinking and meaningful learning achievement. The strategies used to teach physics at this time looking for the transposition of the acquired knowledge to other contexts, thereby facilitating adequate approach to problem situations that might include some experience associated with the solution of these. Most studies involving the search for solutions to these problematic situations are addressed in the context of the teaching of optics, including the use of the Fourier transform, into their applications can stand the theory of diffraction, resonators theory, obtaining optical and digital of the fractional Fourier transform and finally the correlation operation [3].

From this perspective, in this research, we propose an alternative educational way for the construction of the concepts of Fresnel and Fraunhofer diffraction, using the Fourier transform on the study of electromagnetic wave propagation in free space.

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The method makes use of the transparency operator of curvature the field and sphere Fourier operator in paraxial approximation, in order to facilitate the mathematical approach the topic by changing the educational approach traditionally used, and encouraging the use of creative attitudes by both the teacher and students, helping them to reason and operate the physical concepts with a higher level than normally obtained in the study of physical phenomena such.

2. FRESNEL DIFFRACTION AND FOURIER SPHERE OPERATOR

To display the alternative teaching approach since the concept of diffraction, consider Figure 1, which shows an input plane $U_A(\xi,\eta)$ called the diffraction plane and $U_P(u,v)$. The output plane is called observation plane. The finite limits of the aperture have been incorporated in the definition of $U_A(\xi,\eta)$, and the input is considered uniformly illuminated by monochromatic plane wave with unit amplitude and normally.

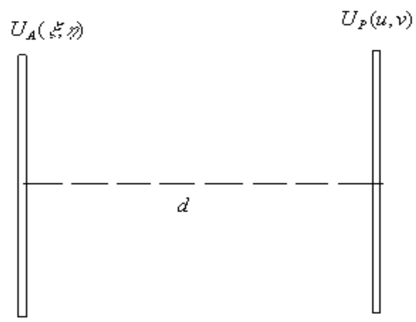

Figure 1. Free space propagation

To demonstrate the use of alternative teaching approach applied to the concept of diffraction, consider Figure 1, which shows an input plane $U_A(\xi,\eta)$ called the diffraction plane and $U_P(u,v)$, the output plane; called observation plane. The finite limits of the aperture function have been included in the definition of $U_A(\xi,\eta)$, and the input is considered uniformly illuminated by a monochromatic plane wave normally incident unit amplitude; applying the Fresnel diffraction integral we obtain the following expression:

$$U_P(u,v) = \frac{1}{i\lambda d}\exp\left(\frac{-i\pi(u^2+v^2)}{\lambda d}\right) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left(\frac{-i\pi(\xi^2+\eta^2)}{\lambda d}\right)\exp\left(\frac{2i\pi(u\xi+v\eta)}{\lambda d}\right)U_A(\xi,\eta)\,d\xi\,d\eta \tag{1}$$

The origin of time P is exchanged to the origin of time A, then the factor

$$\exp\left(\frac{-2i\pi d}{\lambda}\right) \tag{2}$$

Has been neglected. Accordingly, the Fresnel diffraction equation can be written as follows:

$$U_P(u,v)\exp\left(\frac{i\pi(u^2+v^2)}{\lambda d}\right) = \frac{1}{i\lambda d}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U_A(\xi,\eta)\exp\left(\frac{-i\pi(\xi^2+\eta^2)}{\lambda d}\right)\exp\left(\frac{2i\pi(u\xi+v\eta)}{\lambda d}\right)d\xi d\eta \tag{3}$$

But the multiplication of the complex amplitude and phase factor can be interpreted in the paraxial approximation equal to the complex amplitude distribution of spherical surfaces with a radius d for the input and radio $-d$ for the output. Therefore can be rewritten as:

$$U_{A_{Sph}}(\xi,\eta) = U_A(\xi,\eta)\exp\left(\frac{-i\pi(\xi^2+\eta^2)}{\lambda d}\right) \tag{4}$$

$$U_{P_{Sph}}(u,v) = U_P(u,v)\exp\left(\frac{-i\pi(u^2+v^2)}{\lambda(-d)}\right) \tag{5}$$

The equation (4) and (5) means that the complex amplitude distributions are geometrically spherical in shape for both the diffraction surface $U_{A_{Sph}}(\xi,\eta)$ and the surface of observation $U_{P_{Sph}}(u,v)$; also shows that the spherical surfaces are tangent to the respective planes. Therefore, the Fresnel diffraction can be written as:

$$U_{P_{Sph}}(u,v) = \frac{1}{i\lambda d}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U_{A_{Sph}}(\xi,\eta)\exp\left(\frac{2i\pi(u\xi+v\eta)}{\lambda d}\right)d\xi d\eta \tag{6}$$

Simultaneously, equation (6) can be transformed into:

$$U_{P_{Sph}}(u,v) = \frac{2\pi}{i\lambda d}\Im\left[U_{A_{Sph}}(\xi,\eta)\right] \tag{7}$$

Where $\Im$ is a conventional Fourier transform.

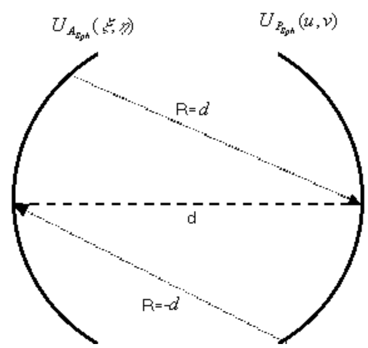

Figure 2. Fresnel diffraction between the spherical surfaces

Thus, the Fresnel diffraction is a Fourier transform between two spherical surfaces where the vertices are located in the same axis and separated by a distance similar to the radius of the sphere. This result allows to define the Fourier sphere operator. Consequently $U_{P_{Sph}}(u,v)$ is the Fourier sphere [4-6] of $U_{A_{Sph}}(\xi,\eta)$. Obviously, this interpretation complies with the Huygens principle, ie the spherical surface of the radio transmitter that propagates *d* be observed at a distance *d*, but on a spherical surface of the radio receiver *-d*.

Thus, ***"If A_{Sphe} is the emitter spherical with radius R = d, and P_{Sphe} is a receiver spherical with radius R =-d and both form a concentric surface. The field that is transferred from A_{Sphe} to P_{Sphe} corresponds to Fraunhofer diffraction phenomenon and is mathematically expressed as a Fourier transform."***

3. CURVATURE TRANSPARENCY OPERATOR

Considering the geometry of Figure 3, assume that a spherical monochromatic wave is illuminating the left side of a positive lens.

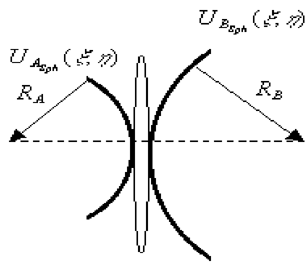

Figure 3. Curvature transparency operator

To calculate the complex amplitude distribution of the field on the right side of the lens $U_B(\xi,\eta)$, considering Figure 3, and using properties of the Fourier transform of a lens, one can show that:

$$U_B(\xi,\eta) = U_A(\xi,\eta)\exp\left(\frac{2i\pi\Delta_o}{\lambda}\right)\exp\left(\frac{-i\pi(\xi^2+\eta^2)}{\lambda}(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\right) \tag{8}$$

And assuming that $R_1 = R_B$ and $R_2 = R_A$ the amplitude distribution on the right side of the lens becomes

$$U_B(\xi,\eta)\exp\left(\frac{-i\pi(\xi^2+\eta^2)}{\lambda}\left(\frac{1}{R_B}\right)\right) = U_A(\xi,\eta)\exp\left(\frac{2i\pi\Delta_o}{\lambda}\right)\exp\left(\frac{-i\pi(\xi^2+\eta^2)}{\lambda}\left(\frac{1}{R_A}\right)\right)\exp\left(\frac{-i\pi(\xi^2+\eta^2)}{\lambda}(n)\left(\frac{1}{R_B}-\frac{1}{R_A}\right)\right) \tag{9}$$

In equation (9), the factor $\exp\left(\frac{-i\pi(\xi^2+\eta^2)}{\lambda}\left(\frac{1}{R_B}\right)\right)$ is a quadratic phase factor, represented by a quadratic approximation to a spherical wave converging to a point of light at a distance R_B .

Similarly, the factor $\exp\left(\frac{-i\pi(\xi^2+\eta^2)}{\lambda}\left(\frac{1}{R_A}\right)\right)$ is a quadratic phase factor, represented by a quadratic approximation to a spherical wave diverging light toward a midpoint in a distance R_A .

Then equation (9) is given as:

$$U_{B_{Sph}}(\xi,\eta) = U_{A_{Sph}}(\xi,\eta)\exp\left(\frac{2i\pi\Delta_o}{\lambda}\right)\exp\left(\frac{-i\pi(\xi^2+\eta^2)}{\lambda}(n)\left(\frac{1}{R_B}-\frac{1}{R_A}\right)\right) \tag{10}$$

If $\Delta_o \neq 0$, it is assumed that the lens has a thickness and the spherical surfaces coincide with the surfaces of the faces of the lens. Where $U_{B_{Sph}}$ is the complex amplitude distribution of radio R_B and $U_{A_{Sph}}$ is the amplitude distribution is the spherical surface with radius R_A .

In the case where $\Delta_o \to 0$ transparency of curvature operator is obtained. Then:

$$U_{B_{Sph}}(\xi,\eta) = U_{A_{Sph}}(\xi,\eta)\exp\left(\frac{-i\pi(\xi^2+\eta^2)}{\lambda}\left(\frac{1}{R_B}-\frac{1}{R_A}\right)\right) \tag{11}$$

Considering two spherical segments *A* and *B* with radii R_A and R_B shown in Figure 3, states that the transfer from the emitter field spherical A_{Sph} tangential to a spherical surface B_{Sph} is expressed by a quadratic phase factor dependent and radii of curvature A_{Sph} and B_{Sph}.

4. EMITTER A RECEIVER TRANSFER

Using the described operators, equation electromagnetic field transfer between a spherical emitter *A* and spherical receiver *B* separated by a distance *d* can be obtained with complete generality (see Figure 4).

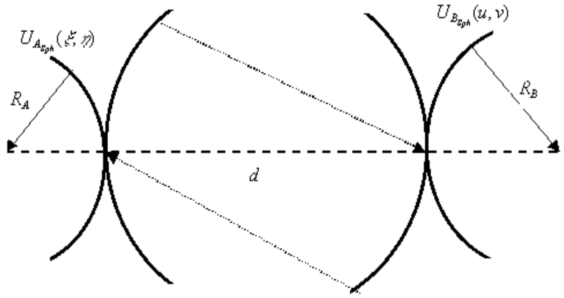

Figure 4. Emitter and receiver system.

With successive applications of two operators, one can achieve the electromagnetic transfer, with complete generality between transmitter R_A and receiver R_B separated by a distance *d* (see Figure 4.). Therefore:

$$U_{B_{Sph}}(u,v)=\frac{2\pi}{i\lambda d}\exp\left(\frac{-i\pi\left(u^2+v^2\right)}{\lambda}\left(\frac{1}{R_B}+\frac{1}{d}\right)\right)\Im\left[U_{A_{Sph}}(\xi,\eta)\exp\left(\frac{-i\pi\left(\xi^2+\eta^2\right)}{\lambda}\left(\frac{1}{d}-\frac{1}{R_A}\right)\right)\right] \quad (12)$$

In the special case of diffraction from a plane screen located at a finite distance, when $R_A \to \infty$ and $R_B \to \infty$, the Fresnel diffraction formula is achieved; for the case $R_A \to \infty$ or $R_B \to \infty$ a finite distance, ie spherical transmitter or receiver, an appropriate scaling the coordinates allows obtaining a fractional Fourier transforn; finally, when $R_A \to \infty$, $R_B \to \infty$ and $d \to \infty$, the resulting expression corresponds to the known phenomenon of Fraunhofer diffraction[7].

CONCLUSIONS

The analysis proposed in this work can prove, that the passage of a spherical wave with a radius of curvature well known through a thin lens and the spherical wave study, corresponds mathematically transparent curvature operator. Furthermore, it was found that the Fresnel diffraction metaxial approximation corresponds mathematically to the Fourier transform between spherical surfaces, reaching the sphere of Fourier operator. Furthermore, it is important to note that the composition of the optical operators allows a more simple way to study the propagation of electromagnetic waves in free space, that is, with the application of the two operators, it is possible to obtain the propagation of the electromagnetic field more generally from sender to receiver, when they are separated by an arbitrary distance given. This last aspect, suggests that the proposal submitted, using the transparency of curvature operators and Fourier sphere to study wave propagation in free space, facilitates the construction of the concepts of Fresnel diffraction and Fraunhofer of more effectively and easily than traditional teaching, emphasizing its educational role.

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