

Studying chaos in carbon-dioxide lasers in a graduate level optical laboratory

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ABSTRACT

Carbon-dioxide lasers are well-known example of optical systems , which can exhibit complex transitions from the CW to the chaotic regime in the intensity caused by the modulation of various system parameters. They can demonstrate classical bifurcation scenarios with period doubling, bistability, and chaotic attractors .

We report on a new method of observing chaos in carbon-dioxide lasers where the simple technique of modulation of the optical feedback parameters is applied. In this approach the parameters of the carbon-dioxide laser itself remain unchanged and modulation is carried out externally. This leads to a system which can be easily assembled and studied in a graduate laboratory. Experimental results show the presence of stable periodic orbits of different periods, bistability of periodic orbits and chaotic attractors with complex transition scenarios, providing a wealth of examples for the student to investigate.

Key words : carbon-dioxide laser, chaos, period doubling, bifurcations, optical feedback, CO₂ laser parameter modulation, periodic orbit, attractor, bistability

1. INTRODUCTION AND BACKGROUND

The transition from regular to irregular behavior in nonlinear dynamical optical systems has been a topic of both experimental and theoretical interest for over two decades. The observation of nonlinear dynamical phenomena and chaos has become a very important issue during last years from both a scientific and practical point of view and thus must be a subject of education in optics . In the context of laser systems one is interested, for example, in the practical problem of the control of the onset of chaos and in the decrease of dynamical noise in the laser output intensity.

It is well known that CO₂ lasers can demonstrate classical transition scenarios to chaos (such as period doubling cascade, for instance) and therefore are a very good system for studying chaos . To investigate the chaos onset mechanisms a number

of laser configurations in which chaos is induced in the output have been studied, e.g., a CO₂ laser with internal harmonic modulation of the loss parameter¹⁻⁴, CO₂ laser with harmonic modulation of the pumping current^{5,6}, a single CO₂ lasers with delayed opto-electronic feedback⁷⁻⁹, and two or more optically coupled lasers^{6,10,11}.

It is also known that CO₂ lasers are very sensitive to radiation feedback. In this work the extreme sensitivity of CO₂ lasers to optical feedback is employed to induce dynamical instabilities. We report here on the experimental and theoretical results concerning instabilities in CO₂ lasers induced by an optical phase modulation of the back reflected signal. The above mentioned phase modulation was implemented by periodically varying either geometrical feedback length or the optical feedback length by means of a refractive index modulation using an electro-optical modulator (EOM). This new scheme of observing chaos in carbon-dioxide lasers doesn't require EOM accommodation inside of laser resonator and thus is significantly simpler than traditional schemes¹⁻⁴. Such scheme can thus be easily implemented in a graduate level optical laboratory.

2. EXPERIMENTAL PROCEDURE

A schematic of the experimental setup is shown in Fig.1. In this scheme an axial

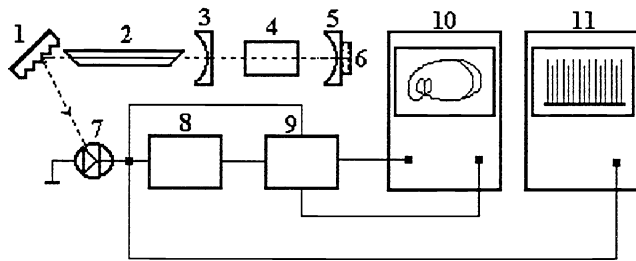


Figure 1

Experimental setup for obtaining chaos in a CO₂ laser: 1. diffraction grating; 2. CO₂ laser tube; 3. output mirror; 4. electro-optical modulator (EOM); 5. additional mirror; 6. piezo-ceramic drive; 7. liquid nitrogen cooled HgCdTe detector; 8. delay line; 9. digitizer; 10. oscilloscope; 11. spectrum analyzer.

flowing gas CO₂ laser operating at a wave length of $\lambda = 10.6 \mu\text{m}$ was used. This laser has a resonant cavity 2.5m long formed by a diffraction grating (1) and mirror (3), the diameter of the discharge tube (2) is 6 mm, and the pressure of the conventional gas mixture varies from 15 to 20 mmHg. An additional mirror (5) provides the radiation feedback into the active region. Mirror (5) is mounted on a piezoelectric ceramic drive (6) allowing static tuning and/or periodic modulation of the external resonator length. The distance between mirrors (3) and (5) does not exceed 30 cm. The laser intensity is measured by a HgCdTe detector (7) which monitor part of the radiation reflected from the diffraction grating (1). The intensity signal from the detector and its time-delayed copy are fed into an oscilloscope (10) operating in the x-y mode. The delay line (8) has a constant delay of 2.5 μs . This simple scheme allows real time observations of the two-dimensional projections of the system trajectories in the space of delayed signals.

We propose to organize the experiments into two major series following the approach we used when working with the system.

In the first series of experiments, (Section 3.1), the optical phase of the back-reflected emission was modulated by using an electro-optical modulator (EOM) with a quarter wave voltage of 2.2 KV. The EOM was a Cd-Te crystal having an active length of 48 mm, and an aperture diameter of 3 mm. The voltage applied to the EOM had a DC component of 1.5 KV and the amplitude and frequency of the modulated signal varied within 100-900V and 50-85 KHz, respectively.

In the second series of experiments, discussed in Section 3.2, the phase of the feedback radiation was modulated by the induced oscillations of the feedback providing mirror (5) mounted on a piezoelectric drive. The driving frequencies varied from 30 KHz to 60 KHz. The frequency range of these drives is centered on the relaxational frequencies of the laser system used. The largest amplitude of the external mirror oscillations was of the order of 1 μm for the lowest driving frequencies and estimated as not exceeding 0.5 μm for the highest driving frequencies used. In both series of experiments the amount of optical feedback radiation was measured to be less than 10%.

3. EXPERIMENTAL RESULTS

3.1 An optical phase modulation by means of EOM

The results presented in Fig. 2 were obtained by varying the static part of the loss coefficient while all other laser parameters remained unchanged. The static part of the loss coefficient was varied by changing the angle of the output mirror of the laser. In the right column of Fig. 2 the frequency spectra are displayed, the middle column shows the respective phase portraits (Intensity(t) vs Intensity(t+ τ)) and finally the left column contains the corresponding time series. These spectra were recorded at a constant frequency of 83 KHz and a modulation amplitude of 150 V.

For relatively low levels of losses the laser intensity follows the driving signal with

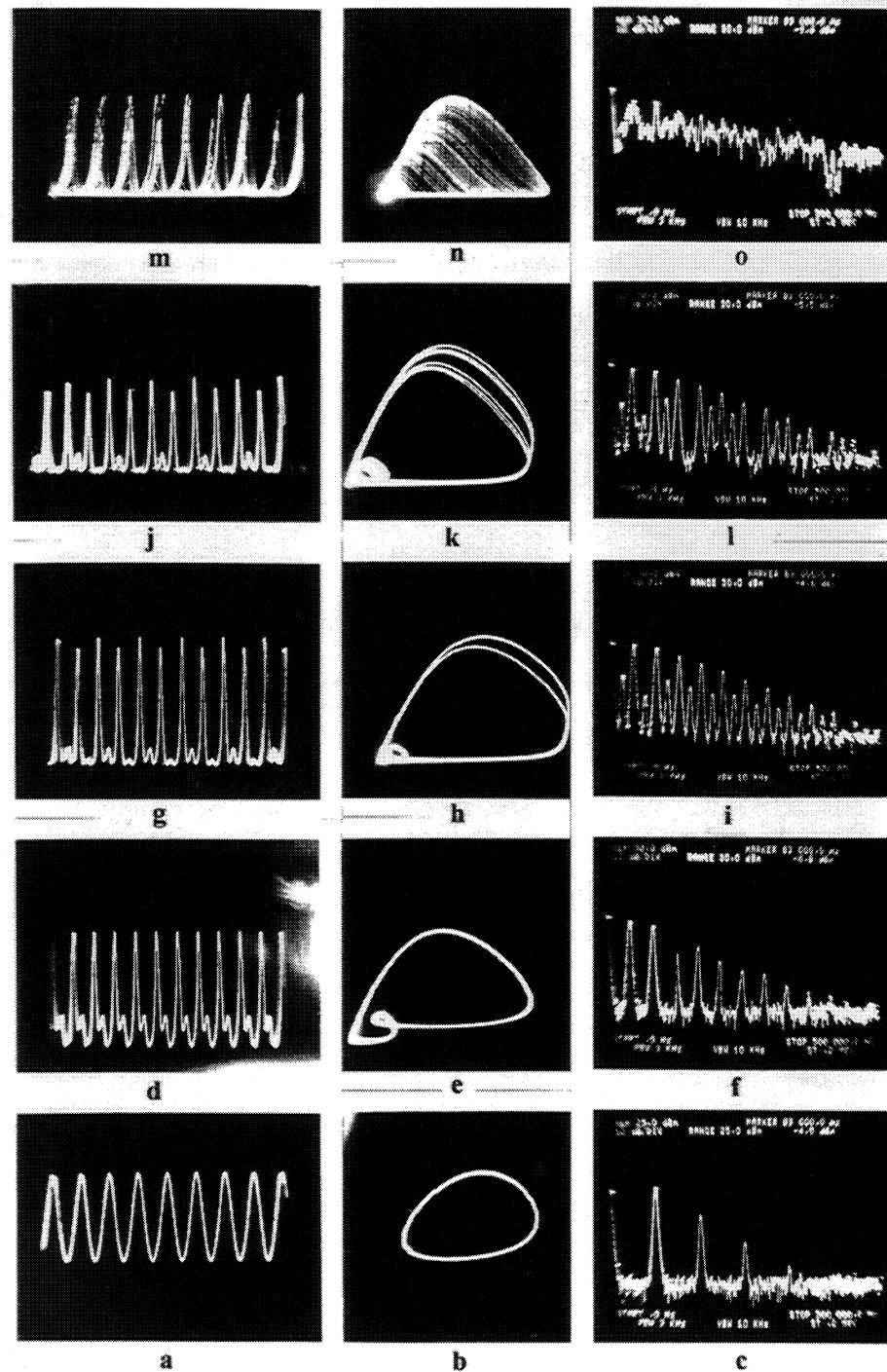


Figure 2

A representative sequence of laser states for a CO_2 laser obtained by varying the static part of the loss coefficient while all other laser parameters were unchanged.

a pronounced second and third harmonic present (Fig. 2c). As the losses are increased, the laser output becomes more and more structured and period 2 oscillations are observed as shown in Figs. 2d,e,f. Subsequently, period 4 is obtained for increasing losses as shown in Figs. 2g,h,i. Similar results were obtained for period 8 Figs. 2j,k,l. Finally the frequency spectrum of the laser output changes into a broad spectrum with a rise in the floor level. This is a definitive indication that the system is driven into chaos, as shown in Figs. 2m,n,o. The chaotic attractor (Fig. 2n) has a characteristic shape typical for chaotic states obtained at other driving frequencies.

The bifurcation diagram presented in Fig. 3 was obtained by varying the amplitude

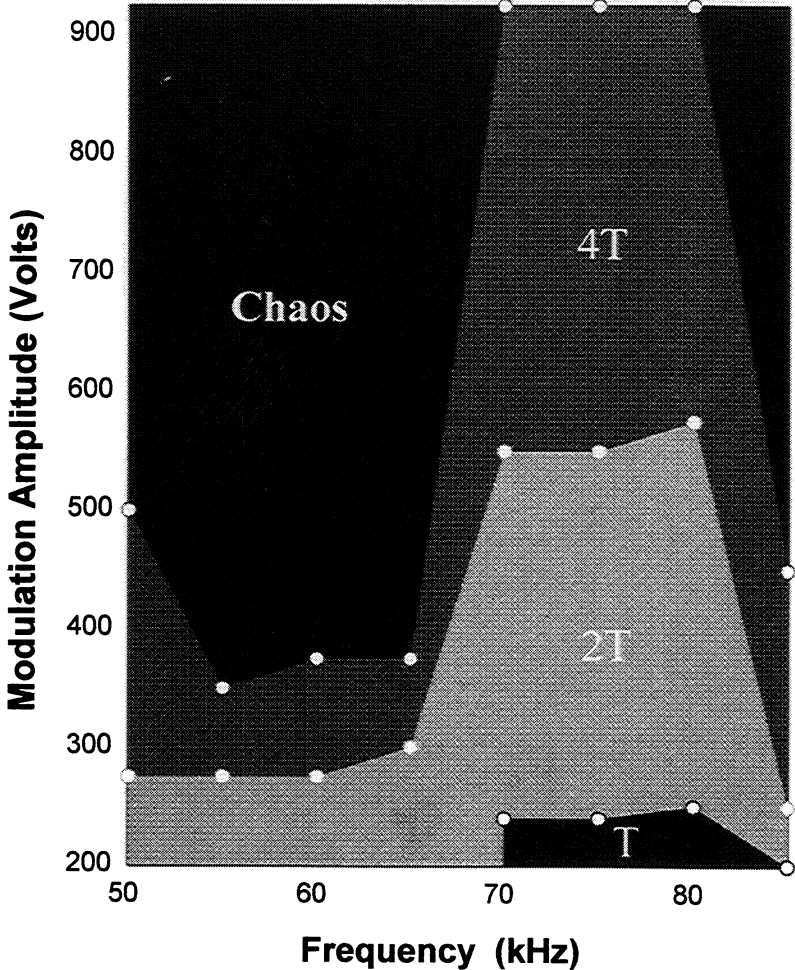


Figure 3

A bifurcation diagram in the (driving frequency, modulation amplitude) parameter space for a CO₂ laser with a phase driven by an electro-optical modulator and with a fixed static part of the intensity loss coefficient.

of ac voltage driving the EOM at a given frequency. The sequence of period doubling bifurcation obtained changing the amplitude of ac voltage, mentioned above, was identical to that obtained by varying the static part of the loss coefficient as described in the previous paragraph. In this case the modulation amplitude was varied while keeping all other parameters of the laser constant. The results obtained by this method are presented in Fig. 3 in the form of a complete bifurcation diagram. The bifurcation diagram clearly indicates that there is a broad range of frequencies in which the laser system could be driven into instability and chaos.

The coexistence of two stable periodic orbits is shown in Fig. 4. Transitions between two orbits of different periods are shown in Figs. 4 a,b. The corresponding frequency spectrum (Fig.4b) contains the fundamental frequencies of both orbits. Similar bistabilities were reported previously^{1,3} with transitions between orbits of period 's 3 and 4.

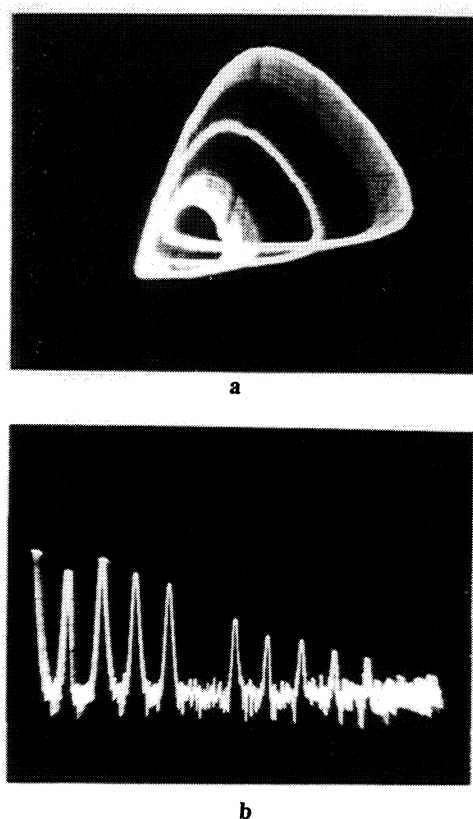
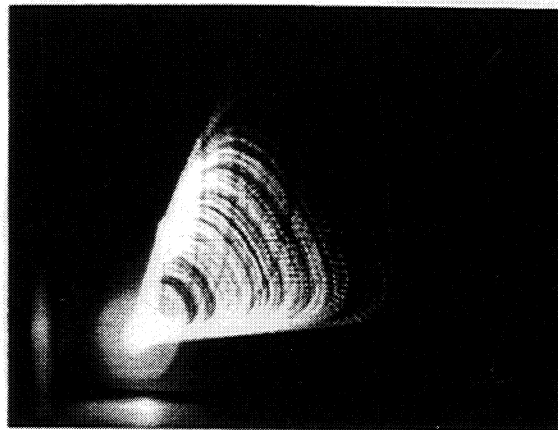


Figure 4

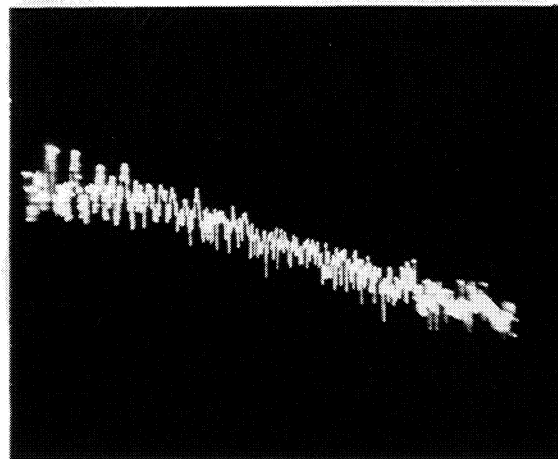
Bistability in CO₂ laser with externally modulated optical feedback.
 EOM is used for optical phase modulation :
 a- phase portrait, b-spectrum ; driving frequency $f_d = 84$ KHz.

3.2. An optical feedback length modulation by means of vibrating mirror

Instabilities and chaos induced by a vibrating mirror are presented in Fig. 5. The period doubling sequence, bifurcations, and attractor shapes obtained in this series of tests are very similar to those presented in section 3.1 for the optical phase modulation by EOM.



a



b

Figure 5
Chaos induced by a vibrating mirror ,
a- phase portrait; b- spectrum, $f_d = 30$ KHz.

Fig.6 shows the coexistence of two period 1 orbits of different amplitudes and the same frequency. This is evidenced by the frequency spectrum shown in Fig. 6b with fundamental frequency and higher harmonics.

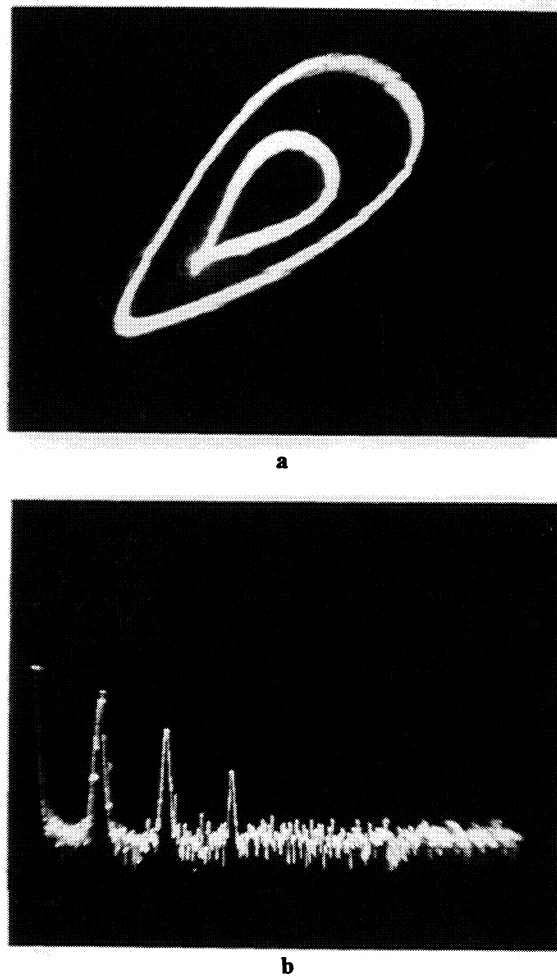


Figure 6
Bistability in CO₂ laser with modulating length of feedback by vibrating mirror
a- phase portrait, b-spectrum, $f_d = 60$ KHz.

4. SUMMARY AND CONCLUSIONS

We have shown that a CO₂ laser can be driven into chaotic state by using a very simple method where one modulates externally the feedback of laser radiation.

Two approaches were used to drive the laser into chaotic state. In the first setup an EOM is used to modulate optical phase of re-injected signal, and the second one modulating the length (and, of course, the phase) of the optical feedback. Both of these methods produced instabilities and chaos through period doubling cascades for a broad range of modulation frequencies. Under certain conditions the system also exhibits bistability. Both methods of chaos observation in carbon-dioxide laser can be successfully used for many types of low power CO₂ lasers to study of nonlinear dynamical phenomenon in a graduate student laboratory.

The results of this work are very similar to those obtained for a CO₂ laser with modulated intercavity losses¹⁻⁴. This is not coincidental, since it can be shown that the rate equations of class B laser with externally modulated and re-injected optical signal (see Appendix) are equivalent to ones describing a system with modulated losses. However, our external modulation schemes are much simpler experimentally than the intercavity modulation and thus better suited for a graduate laboratory.

5. ACKNOWLEDGEMENTS

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6. APPENDIX

6.1 Rate Equations of a Class B Laser with Externally Modulated and Re-Injected Optical Signal

CO₂ lasers belong to the so-called B-class of lasers where the polarization variable has the shortest relaxation time γ_L^{-1} of the system. The relaxation time k^{-1} of the field variable is typically three orders of magnitude longer than γ_L^{-1} in this class of lasers, whereas the relaxation time γ_I^{-1} of the population inversion variable sets up the longest intrinsic time scale, that is $\gamma_I^{-1} \geq k^{-1} \gg \gamma_L^{-1}$. This hierarchy of relaxation times allows the adiabatic elimination of the polarization variable from the Maxwell-Bloch equations¹². In the single mode case, these systems are reasonably well described by the rate equations for the field amplitude E and the population inversion ΔN :

$$\frac{dE}{dt} = -kE(1+i\theta') + \frac{g^2}{\gamma_{\perp}} \frac{\Delta NE}{1+i\delta'} + krE \exp[i\varphi(t)] \quad (1)$$

$$\frac{d(\Delta N)}{dt} = -\gamma_{\parallel}(\Delta N - \Delta N_o) - 2 \frac{2g^2}{\gamma_{\perp}} \frac{1}{1+\delta'^2} |E|^2 \Delta N \quad (2)$$

where r - is the reflection coefficient of an external mirror;
 $r E \exp[i\varphi(t)]$ - represents re-injected field with time depending phase $\varphi(t)$;
 $\delta' = (\omega_o - \omega_L) / \gamma_{\perp}$; $\theta' = (\omega_c - \omega_L) / k$; ω_c , ω_o , and ω_L are the cavity, atomic, and laser operational frequencies, respectively.

Equations for the light intensity I and population inversion ΔN can be obtained from eq. (1) and (2) assuming that time depending phase $\varphi(t)$ can be described as $\varphi(t) = \varphi_o + \varphi_m \cos \Omega t$, where: φ_o - constant part of phase; φ_m - phase modulation amplitude; Ω - modulation frequency.

$$\frac{dI}{dt} = GI\Delta N - Ik_o [1 + m \cos(\Omega t)] \quad (3)$$

$$\frac{d\Delta N}{dt} = -\gamma_{\parallel}(\Delta N - \Delta N_o) - 2GI\Delta N, \quad (4)$$

where : $G = 2g^2 / [\gamma_{\perp}(1+\delta'^2)]$; $k_o = 2k(1-r \cos \varphi_o)$;
 $m = (r \varphi_m \sin \varphi_o) / (1-r \cos \varphi_o)$.

Equations (3) and (4) are identical to appropriate equations for light intensity and population inversion for system with modulated losses¹.

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